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# BULLETIN

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### ON AN INFINITE SERIES OF INTEGRALS INVOLVING STURM—LIONVILLE EIGEN FUNCTIONS

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#### INTRODUCTION.

In a previous paper<sup>1</sup> it has been proved by the writer that the series

$$\frac{\sum_n |a_{k,j}^{(n)}|}{k_1^2 k_2^2 \dots k_r^2 j_1^3 j_2^3 \dots j_s^3}, \text{ where}$$

$$a_{k,j}^{(n)} = j_1 j_2 \dots j_s \int_0^\pi \sin k_1 x \dots \sin k_r x \cos j_1 x \dots \cos j_s x \sin n x \, dx$$

is uniformly convergent for all  $k_r, j_s$

Now  $\sin n x$  is just the Eigen function of a particular differential equation, viz.  $\frac{d^2 y}{dx^2} + \lambda y = 0$  for certain boundary values, viz.  $y(0) = 0, y(\pi) = 0$ , and it is natural to ask, therefore, whether a similar property is possessed by other Eigen-functions.

In another paper<sup>2</sup> the writer has shown that if  $\lambda_n (n=1, 2, \dots)$  are the Eigen-values and  $\phi_n(x)$  the corresponding Eigen-functions of the Sturm-Lionville Differential Equation

$$\frac{d}{dx} \left( p \frac{dy}{dx} \right) + \lambda y = 0$$



with the boundary conditions  $y(0)=0$ ,  $y(\pi)=0$ , and if we define

$$a_{j,k}^{(n)} = \int_0^\pi \phi_j(x) \phi_k(x) \phi_n(x) dx,$$

then the series  $\frac{\sum |a_{j,k}^{(n)}|}{\lambda_j \lambda_k}$  is uniformly convergent.

In the present paper, we prove the more general theorems that the series

$$\sum_{n=1}^{\infty} \frac{\sqrt{\lambda_n} |a_{j,k}^{(n)}|}{\lambda_j \lambda_k},$$

and

$$\sum_{n=1}^{\infty} \frac{\sqrt{\lambda_n} |b_{k,j}^{(n)}|}{\lambda_k \lambda_j^{\frac{3}{2}}},$$

also converge uniformly for all  $j, k \geq 1$ , where

$$b_{k,j}^{(n)} = \int_0^\pi \phi_k(x) \frac{d\phi_j}{dx} \phi_n(x) dx.$$

These Theorems are important in the theory of higher partial differential equations, as has been shown by the writer in a paper communicated to the London Math. Society.

# §1.

Let  $p(x)$  be an essentially positive function defined in the interval  $0 \leq x \leq \pi$ , and let  $p(x)$  as well as  $\frac{dp}{dx}$  and  $\frac{d^2p}{dx^2}$  be continuous and uniformly bounded in the whole interval.

Let  $\lambda_n$  be the characteristic values and

$$\phi_n(x) \quad (n=1, 2, 3, \dots)$$

the characteristic functions of the Sturm-Liouville differential equation

$$(A) \quad \frac{d}{dx} \left( p \frac{dy}{dx} \right) + \lambda y = 0$$

with the boundary conditions

$$(B) \quad y(0)=0, \quad y(\pi)=0.$$

We assume that the characteristic functions are orthogonal and normalised.

For any pair of  $j, k$  we define a sequence of functions  $a_{j,k}^{(n)}$  by the relation

$$(1) \quad a_{j,k}^{(n)} = \int_0^\pi \phi_j(x) \phi_k(x) \phi_n(x) dx.$$

(n=1, 2, ...)

and we shall prove the theorem that the series

$$(2) \quad \sum_{n=1}^{\infty} \frac{\sqrt{\lambda_n} |a_{j,k}^{(n)}|}{\lambda_j \lambda_k}$$

is uniformly convergent for all  $j, k$ .

The asymptotic expansions of  $\lambda_n$ ,  $\phi_n(x)$  and  $\frac{d\phi_n}{dx}$  for large  $n$  are known to be<sup>3</sup>

$$(3) \quad \lambda_n = n^2 \frac{\pi^2}{l^2} + O(1), \quad \sqrt{\lambda_n} = n \frac{\pi}{l} + O\left(\frac{1}{n}\right),$$

$$\phi_n(x) = C_n \frac{\sin n p_2(x)}{\sqrt{p}} + O\left(\frac{1}{n}\right),$$

$$\frac{d\phi_n}{dx} = C_n \frac{n\pi}{l} \frac{\cos n p_2(x)}{\sqrt{p}} + O(1),$$

where

$$l = \int_0^{\pi} \frac{1}{\sqrt{p}} dx,$$

$$(4) \quad p_2(x) = \frac{\pi}{l} \int_0^x \frac{1}{\sqrt{p}} dx$$

$$\frac{1}{C_n^2} = \int_0^{\pi} \frac{\sin^2 n p_2(x)}{\sqrt{p}} dx.$$

Now, since  $p(x)$  is essentially positive and  $\sin n p_2(x)$  does not vanish identically in  $0 \leq x \leq \pi$ , we deduce that  $\frac{1}{C_n}$  is greater than a + ve number  $c$  for all  $n$ , and therefore  $C_n$  is bounded. Consequently, the functions  $\phi_n(x)$  and  $\frac{1}{n} \frac{d\phi_n}{dx}$  are also bounded for all  $n$ .

Now, we have

$$a_{j,k}^{(n)} = \int_0^{\pi} \phi_j \phi_k \phi_n dx;$$

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integrating by parts and considering that  $\phi_n$  satisfies the equation (A) and the condition (B), we get:

$$\begin{aligned} a_{j,k}^{(n)} &= -\frac{1}{\lambda_n} \int_0^\pi \phi_j(x) \phi_k(x) \frac{d}{dx} \left\{ p \frac{d\phi_n}{dx} \right\} dx, \\ &= -\frac{1}{\lambda_n} \int_0^\pi \phi_n(x) \frac{d}{dx} \left\{ p \frac{d}{dx} (\phi_j \phi_k) \right\} dx, \\ &= -\frac{1}{\lambda_n} \int_0^\pi \phi_n(x) \left[ \frac{d}{dx} \left( p \frac{d\phi_j}{dx} \right) \phi_k(x) + \frac{d}{dx} \left( p \frac{d\phi_k}{dx} \right) \phi_j \right. \\ &\quad \left. + 2 p \frac{d\phi_j}{dx} \frac{d\phi_k}{dx} \right] dx, \\ &= -\frac{1}{\lambda_n} \int_0^\pi \phi_n(x) \left\{ -(\lambda_j + \lambda_k) \phi_j(x) \phi_j(x) \phi_k(x) \right. \\ &\quad \left. + 2 p \frac{d\phi_j}{dx} \frac{d\phi_k}{dx} \right\} dx. \end{aligned}$$

Since  $\lambda_j, \lambda_k$  are  $> 1$  for all  $j, k$ , therefore we have

$$(5) \quad \frac{a_{j,k}^{(n)}}{\lambda_j \lambda_k} < \frac{2}{\lambda_n} \left\{ |a_{j,k}^{(n)}| + |d_{j,k}^{(n)}| \right\},$$

where

$$(6) \quad d_{j,k}^{(n)} = \int_0^\pi p(x) \frac{1}{\lambda_j} \frac{d\phi_j}{dx} \frac{1}{\lambda_k} \frac{d\phi_k}{dx} \phi_n dx$$

Therefore,

$$\begin{aligned} (7) \quad \left( \sum_n \sqrt{\lambda_n} \left| \frac{a_{j,k}^{(n)}}{\lambda_j \lambda_k} \right| \right)^2 &< \left( \sum_n \frac{2}{\sqrt{\lambda_n}} \left\{ |a_{j,k}^{(n)}| + |d_{j,k}^{(n)}| \right\} \right)^2 \\ &< \sum_n \frac{4}{\lambda_n} \sum_n \left\{ |a_{j,k}^{(n)}| + |d_{j,k}^{(n)}| \right\}^2 \end{aligned}$$

using the inequality of Schwarz. If we use further the inequality  $2ab \geq a^2 + b^2$ , then we get

$$(8) \quad \left( \sum_n \sqrt{\lambda_n} \left| \frac{a_{j,k}^{(n)}}{\lambda_j \lambda_k} \right| \right)^2 < \sum_n \frac{8}{\lambda_n} \sum_n \left\{ |a_{j,k}^{(n)}|^2 + |d_{j,k}^{(n)}|^2 \right\}$$

But we have

$$\begin{aligned} \phi_j(x) \phi_k(x) \phi_n(x) = & \left\{ C_j \frac{\sin j p_2}{\sqrt[4]{p}} + O\left(\frac{1}{j}\right) \right\} \left\{ C_k \frac{\sin k p_2}{\sqrt[4]{p}} + O\left(\frac{1}{k}\right) \right\} \\ & + \left\{ C_n \frac{\sin n p_2}{\sqrt[4]{p}} + O\left(\frac{1}{n}\right) \right\} \end{aligned}$$

and since all the  $C_n$  are bounded uniformly, we can write

$$(9) \quad \int_0^\pi \phi_j(x) \phi_k(x) \phi_n(x) dx < \alpha \int_0^\pi \frac{\sin j p_2 \sin k p_2 \sin n p_2}{p^{\frac{3}{4}}} dx + \beta \frac{1}{j k n},$$

where  $\alpha$  and  $\beta$  are constants properly chosen.

$$\text{Now } p_2(x) = \frac{\pi}{l} \int_0^x \frac{1}{\sqrt{p}} dx,$$

therefore

$$\frac{d p_2}{d x} = \frac{\pi}{l} \frac{1}{\sqrt{p}}, \quad p_2(0) = 0, \quad p_2(\pi) = \pi$$

So that

$$(10) \quad \int_0^\pi \phi_j(x) \phi_k(x) \phi_n(x) dx < \frac{\alpha l}{\pi} \int_0^\pi \frac{\sin j p_2 \sin k p_2 \sin n p_2}{\sqrt{p}} d p_2 + \beta \frac{1}{j k n}.$$

Therefore

$$\begin{aligned} (11) \quad \sum_n (a_{j,k}^{(n)})^2 & < \frac{\alpha^2 l^2}{\pi^2} \sum_n \left( \int_0^\pi \sin j p_2 \sin k p_2 \sin n p_2 \cdot \frac{1}{\sqrt{p}} d p_2 \right)^2 \\ & + \frac{2 \alpha l p}{\pi j k} \sum_n \frac{1}{n} \int_0^\pi \sin j p_2 \sin k p_2 \sin n p_2 \cdot \frac{1}{\sqrt{p}} d p_2 \\ & + \frac{\beta^2}{j^2 k^2} \cdot \sum_n \frac{1}{n^2}. \end{aligned}$$

The series

$$(12) \quad \sum_n \left( \int_0^\pi \sin j p_2 \sin k p_2 \sin n p_2 \cdot \frac{1}{\sqrt{p}} d p_2 \right)^2$$



is uniformly convergent for all  $j, k$ . Moreover,

$$\begin{aligned} & \sum_n \frac{1}{k j n^2} \int_0^\pi \sin j p_2 \sin k p_2 \sin n p_2 \cdot \frac{1}{\sqrt{p}} dp_2 \\ &= \sum_n \frac{1}{k j n^2} \int_0^\pi \cos n p_2 \frac{d}{dp_2} \left\{ \frac{1}{\sqrt{p}} \sin k p_2 \sin j p_2 \right\} dp_2 \\ &= \sum_n \frac{1}{n^2} \int_0^\pi \cos n p_2 \frac{d}{dp_2} \left\{ \frac{1}{\sqrt{p}} \frac{\sin k p_2}{k} \frac{\sin j p_2}{j} \right\} dp_2. \end{aligned}$$

For all values of  $jk$ , the integral on the right is uniformly bounded, and therefore the series

$$(13) \quad \sum_n \frac{1}{k j n} \int_0^\pi \frac{1}{\sqrt{p}} \sin j p_2 \sin k p_2 \sin n p_2 dx$$

is uniformly convergent.

Also for all  $j, k$

$$\frac{1}{j^2 k^2} \sum_n \frac{1}{n^2} \leq \frac{\pi^2}{6}.$$

From (13), We see therefore that the series

$$(14) \quad \sum_n \left( a_{j,k}^{(n)} \right)^2$$

is uniformly convergent for all  $j, k$ .

$$\begin{aligned} \text{Now, } p(x) \phi_n(x) \frac{1}{\lambda_j} \frac{d\phi_j}{dx} \frac{1}{\lambda_k} \frac{d\phi_k}{dx} &= \left\{ C_n p^{\frac{3}{2}} \sin n p_2 + O\left(\frac{1}{n}\right) \right\} \\ &+ \left\{ \frac{C_j}{\pi j} p^{-\frac{1}{2}} \cos j p_2 + O\left(\frac{1}{j^2}\right) \right\} \left\{ \frac{C_k}{\pi k} p^{-\frac{1}{2}} \cos k p_2 + O\left(\frac{1}{k^2}\right) \right\} \end{aligned}$$

So that

$$(15) \quad |a_{j,k}^{(n)}| < \frac{\gamma}{kj} \int_0^\pi p^{\frac{7}{2}} \cos j p_2 \cos k p_2 \sin n p_2 + \delta \frac{1}{k^2 j^2 n^2}$$

where  $\gamma$  and  $\delta$  are two constants properly chosen.

Exactly as in the case of the series  $\sum_n \left( a_{j,k}^{(n)} \right)^2$ , it can be proved that also the series

$$(16) \quad \sum_n \left( d_{j,k}^{(n)} \right)^2$$

is uniformly convergent for all  $j, k$ .

Also, since  $\sum_n \frac{1}{\lambda_n}$  is evidently convergent, we see from (8) on account of (14) and (16) that the series

$$(17) \quad \sum_{n=1}^{\infty} \sqrt{\lambda_n} \frac{1}{\lambda_j \lambda_k} a_{j,k}^{(n)}$$

converges uniformly for all  $j, k$ .

§2.

In this paragraph, we shall prove that on defining the sequence of functions  $b_{k,j}^{(n)}$  more generally by the Integral.

$$(18) \quad b_{k,j}^{(n)} = \int_0^{\pi} \phi_k(x) \frac{d\phi_j}{dx} \phi_n(x) dx$$

a similar theorem holds, viz. that the series

$$(19) \quad \sum_{n=1}^{\infty} \frac{\sqrt{\lambda_n} 1 b_{k,j}^{(n)}}{\lambda_k \lambda_j^{\frac{3}{2}}}$$

converges uniformly for all  $k, j$ .

Since  $\phi_n(x)$  satisfies the differential equation (A) we can write (18) in the form

$$(20) \quad b_{k,j}^{(n)} = -\frac{1}{\lambda_n} \int_0^{\pi} \phi_k \phi'_j \left( p \phi'_n \right)' dx,$$

where dashes are used to denote derivatives. Integrating by parts and remembering that  $\phi_n(x)$  satisfies the boundary conditions (B), we get

$$\begin{aligned} b_{k,j}^{(n)} &= \frac{1}{\lambda_n} \int_0^{\pi} \phi'_n p \left( \phi_k \phi'_j \right)' dx \\ &= \frac{1}{\lambda_n} \int_0^{\pi} \phi'_n \left\{ p \phi'_k \phi'_j - \lambda_j \phi_k \phi_j - p' \phi_k \phi_j \right\} dx. \end{aligned}$$

Integrating again by parts, we have,

$$\begin{aligned}
 b_{k,j}^{(n)} &= -\frac{1}{\lambda_n} \int_0^\pi \phi_n \left\{ \left( p \phi_k' \phi_j' \right)' - \lambda_j \left( \phi_n \phi \right)' - \left( p' \phi_k \phi_j \right)' \right\} dx \\
 &= -\frac{1}{\lambda_n} \int_0^\pi \phi_n \left\{ p' \phi_k' \phi_j' + p \phi_k'' \phi_j' + p \phi_k' \phi_j'' - \lambda_j' \phi_k' \phi_j \right. \\
 &\quad \left. - \lambda_j \phi_k \phi_j' - p'' \phi_k \phi_j' - p' \phi_k' \phi_j' - p' \phi_k \phi_j'' \right\} dx, \\
 &= -\frac{1}{\lambda_n} \int_0^\pi \phi_n \left\{ \phi_j' \left( -\lambda_k \phi_k - p' \phi_k' \right) + \phi_k' \left( -\lambda_j \phi_j - p' \phi_j' \right) \right. \\
 &\quad \left. - \lambda_j \phi_k' \phi_j - \left( \lambda_j + p'' \right) \phi_k \phi_j' + \frac{p'}{p} \phi_k \left( \lambda_j \phi_j + p' \phi_j' \right) \right\} dx, \\
 (21) &= -\frac{1}{\lambda_n} \int_0^\pi \phi_n \left\{ \phi_k \phi_j' \lambda_j \frac{p'}{p} + \phi_k \phi_j' \left( -\lambda_k - \lambda_j - p'' + \frac{p'^2}{p} \right) \right. \\
 &\quad \left. - 2 \lambda_j \phi_k' \phi_j - 2 p' \phi_k' \phi_j' \right\} dx.
 \end{aligned}$$

Now we see that the integrand on the right contains a term  $\lambda_j \phi_n \phi_k \phi_j'$ , and since in the asymptotic expansion of  $\phi_j'$  a factor  $j \frac{\pi}{l}$  also comes, it will be obviously not sufficient to divide this term by  $\lambda_k \lambda_j$  if we want the quotient to be bounded for all  $j$ . On the other hand, a division by  $\lambda_k \lambda_j^{\frac{3}{2}}$  is quite sufficient for this purpose. Thus we have

$$(22) \quad \frac{|b_{k,j}^{(n)}|}{\lambda_k \lambda_j^{\frac{3}{2}}} < \frac{1}{\lambda_n} \left\{ |e_{k,j}^{(n)}| + |f_{k,j}^{(n)}| + 2 |g_{kj}^{(n)}| + 2 |p_{kj}^{(n)}| \right\}$$

where

$$e_{k,j}^{(n)} = \int_0^\pi \frac{p'}{p} \phi_k \phi_j \phi_n dx$$

$$(23) \quad f_{k,j}^{(n)} = \int_0^\pi \left( 2 + p'' + \frac{p'^2}{p} \right) \phi_k \frac{\phi'_j}{\sqrt{\lambda_j}} \phi_n dx,$$

$$g_{k,j}^{(n)} = \int_0^\pi \frac{\phi'_k}{\lambda_k} \phi_j \phi_n dx,$$

$$h_{k,j}^{(n)} = \int_0^\pi p' \frac{\phi'_k}{\lambda_k} \frac{\phi'_j}{\lambda_j} \phi_n dx.$$

Now, it is not difficult to prove exactly as for the series  $\sum_n \left( a_{j,k}^{(n)} \right)^2$  and  $\sum_n \left( d_{j,k}^{(n)} \right)^2$  in the first paragraph, that all the series

$$(24) \quad \sum_n \left( e_{k,j}^{(n)} \right)^2, \quad \sum_n \left( f_{k,j}^{(n)} \right)^2$$

$$\sum_n \left( g_{k,j}^{(n)} \right)^2, \quad \sum_n \left( h_{k,j}^{(n)} \right)^2$$

are uniformly convergent for all  $k, j$ .

From (22) we get on account of Schwarz's inequality

$$(25) \quad \left( \sum_n \frac{\sqrt{\lambda_n} |b_{k,j}^{(n)}|}{\lambda_k \lambda_j^{\frac{3}{2}}} \right)^2 < \sum_n \frac{1}{\lambda_n} \sum_n \left\{ |e_{k,j}^{(n)}|^2 + |f_{k,j}^{(n)}|^2 + 2 |g_{k,j}^{(n)}| + 2 |h_{k,j}^{(n)}| \right\}^2$$

$$< \sum_n \frac{1}{\lambda_n} \sum_n \left\{ 6 |e_{k,j}^{(n)}|^2 + 6 |f_{k,j}^{(n)}|^2 + 12 |g_{k,j}^{(n)}|^2 + 12 |h_{k,j}^{(n)}|^2 \right\}$$

using the inequality  $2ef \leq e^2 + f^2$ , etc.

From (24) and (25) we see therefore that the series

$$(26) \quad \sum_n \frac{\sqrt{\lambda_n} |b_{k,j}^{(n)}|}{\lambda_k \lambda_j^{\frac{3}{2}}}$$

converges uniformly for all  $k, j$ , which proves our theorem.

Finally, we remark that there should be no considerable difficulty in proving the following theorem along similar lines.

If

$$(27) \quad b_{k_1, k_2, \dots, k_r}^{(n)} = \int_0^\pi \phi_n^{(x)} \phi_{k_1} \phi_{k_2} \dots \phi_{k_r} \phi'_{j_1} \phi'_{j_2} \dots \phi'_{j_s} dx$$

$$j_1, j_2, \dots, j_s$$

$$(n=1, 2, \dots)$$



then the series

$$(28) \quad \sum_n \frac{\sqrt{\lambda_n} \left| b_{j_1, \dots, j_s}^{(n)} \right|}{\lambda_{k_1} \lambda_{k_2} \dots \lambda_{k_r} \lambda_{j_1}^{\frac{3}{2}} \lambda_{j_2}^{\frac{3}{2}} \dots \lambda_{j_s}^{\frac{3}{2}}}$$

is uniformly convergent for all  $k_r$  and  $j_s$ .

#### References.

<sup>1</sup> M. R. Siddiqi, "Zur Theorie der nicht-linearen partiellen Differential-gleichungen vom parabolischen Typus," *Math. Zeitschrift*, 35, 475 (11), 1931.

<sup>2</sup> M. R. Siddiqi, On an infinite system of non-linear integral equations. *Bull. Cal. Math. Soc.*, 24, pp. 47-51, 1932.

<sup>3</sup> Courant—Hilbert: *Methoden der mathematische Physik I*, 2nd Ed (1931), pp. 290-291.

<sup>4</sup> M. R. Siddiqi, "Zur Theorie der nicht-linearen partiellen Differential-gleichungen vom parabolischen Typus," *Math. Zeitschrift*, 35, 476 (16), 1931.

# A CLASS OF DIRICHLET'S SERIES POSSESSING ESSENTIAL CHARACTERISTICS OF A TAYLOR'S SERIES

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1. In case of a Taylor's series its circle of convergence is also its circle of absolute convergence, and contains at least one singular point of the function represented by the series. As is well-known, no such simple relation holds in the case of general Dirichlet's series. Indeed a Dirichlet's series convergent in a portion of the plane may be absolutely convergent in a smaller region, and may represent a function all over the plane or in a wider region of it. For example, in the case of the series  $\sum_{n=1}^{\infty} (-1)^{n-1} n^{-s}$ ,  $\sigma_0 = 0$ ,  $\sigma = 1$ , and the function represented by the series is an integral function of  $s$ . There is, however, one important class of Dirichlet's series, namely, the series all of whose coefficients are positive, for which the line of convergence is also the line of absolute convergence, and contains at least one singularity of the function represented by the series.<sup>1</sup> My main object in this paper is to point out the existence of another class of Dirichlet's series for which the lines of convergence and absolute convergence coincide and necessarily contain at least one singularity of the function represented by the series. The result obtained in this direction is embodied in the following theorem.

## 2. THEOREM I.—If

(2.1)  $\lambda(z)$  be a branch of an analytic function of  $z (=x+iy=\rho e^{i\phi})$ ,  $p-1 < \beta < p$  in the angle  $|\phi| \leq \alpha$ ,  $\alpha > 0$ , and  $\lambda(z) = o(\rho)$  uniformly in this angle as  $\rho \rightarrow \infty$ ;

(2.2)  $\lambda(x)$  be an  $L$ -function<sup>2</sup> such that it is positive for  $x \geq p$  and steadily tends to infinity with  $x$ ;

(2.3)  $\lambda'(z) = O(1)$  as  $\rho \rightarrow \infty$  uniformly in the angle  $|\phi| \leq \alpha$ ;

(2.4)  $f(\xi)$  be an analytic function of  $\xi = re^{i\theta}$  in the angle  $|\theta| \leq \alpha$ ,  $\alpha_1 > 0$ , and satisfy the relation  $f(\xi) = O(e^{Mr})$ , throughout this angle;

Also  $|F(s + \rho e^{ia})| < K_2 e^{(M+|s|)|\lambda(z)|} < K_2 e^{(M+|s|)\epsilon\rho}$ , by virtue of hypotheses (2.1), (2.3), (2.4), where  $\epsilon$  is an arbitrarily small positive number and  $\rho$  is sufficiently large.

Hence the integral (2.11) converges like

$$\int_0^\infty e^{-\rho(2\pi \sin \alpha - (M+|s|)\epsilon)} d\rho$$

which is uniformly convergent for all bounded values of  $s$ ,  $\epsilon$  being arbitrarily small.

Similarly we can prove that  $I_3$  represents an integral function of  $s$ .

As regards the integral  $I_1$ , it is equal to

$$\int_0^\infty \lambda'(x) f\{\lambda(x)\} e^{-s\lambda(x)} dx = \int_0^\infty f(\xi) e^{-s\xi} d\xi + \text{an integral function of } s.$$

So that

$$(2.12) \quad H(s) - J(s) = G(s),$$

where  $G(s)$  is an integral function of  $s$ , and  $J(s)$  is an analytic function of  $s$

defined initially by the integral  $\int_0^\infty f(z) e^{-sz} dz$ .

The equation (2.12) has been obtained on the assumption that  $s$  is real, positive and sufficiently large. But as the right-hand side represents an integral function of  $s$  the equation persists for all values of  $s$ . That is, *the finite singularities of  $H(s)$  are identical with those of  $J(s)$ .*

Now by virtue of a result<sup>3</sup> established by me and Mr. Jain recently,  $J(s)$  is an analytic function of  $s$  in the region lying exterior to a convex curve  $\Sigma$  which is the envelope of the lines (2.8) each of which contains at least one singularity of  $J(s)$ . The same is, therefore, true of  $H(s)$ , and in particular

$$\sigma = \sigma_0 = \lambda(0) = \lim_{r \rightarrow \infty} \frac{\log |f(r)|}{r}, \text{ and the line of convergence contains at least}$$

one singularity of  $H(s)$ .

If  $\lambda(\theta) = -\infty$  for any value of  $\theta$ ,  $J(s)$  is an integral function of  $s$ , and so is  $H(s)$ .

This completes the proof of our theorem.

#### REMARKS

3. Theorem I enables us to study the singularities of a class of Dirichlet's series in terms of those of the Laplace-Abel integral. Now we propose to make a few observations on this theorem.

(a) The function  $\lambda(z)$  contemplated in the theorem may be any one of a class of functions such as  $\frac{z}{\log z}$ ,  $\frac{z}{\log \log z}$ ,  $z^a$  ( $0 < a < 1$ ),  $\sqrt{z} \log z$ ,  $(\log z)^\beta$ ,  $(\log \log z)^k$ , etc. It will be noticed that the hypotheses (2.1), (2.2), (2.3) and (2.5) are all true for such a  $\lambda(z)$ . If  $\lambda(z) = z$ , we have the case of a Taylor's series in  $e^{-s}$ . If  $\frac{\lambda(n)}{n} \rightarrow \infty$  as  $n \rightarrow \infty$ , the line of convergence of a Dirichlet's series is, in general, a singular line.

(b) In the coefficient of the  $n$ th term  $\lambda'(n)$  is an essential factor for the truth of the theorem. Suppose  $\lambda(z) = \log \log z$ , and  $f(z) = 1$ . Then the theorem fails for the series  $\sum f\{\lambda(n)\} e^{-s\lambda(n)}$ , but holds for  $\sum \lambda'(n) f\{\lambda(n)\} e^{-s\lambda(n)}$ .

(c) If  $f(z)$  is an integral function of  $s$  of exponential type, then  $H(s)$  is an analytic function of  $s$  outside a closed convex curve  $\Sigma$ . In particular if  $f(z) = \sum_0^\infty \frac{c_\nu z^\nu}{\nu!}$  and the series  $\sum_0^\infty c_\nu z^\nu$  has the circle of its convergence  $|z| = \frac{1}{k}$  as a singular line, then the circle  $|s| = k$  is a singular line for the

function  $H(s)$ , since  $H(s) = G(s) + \sum_0^\infty \frac{c_\nu}{s^{\nu+1}}$ .

Suppose  $H(s) = \sum \frac{\sin \{\pi \log \log n\}}{n \log n} e^{-s \log \log n}$ . Then  $H(s)$

has singular points at  $s = \pm \pi i$ , and with the possible exception of the points lying on the line joining these two singular points  $H(s)$  is analytic everywhere else in the plane.

(d) If  $f(z)$  is analytic in the half-plane  $R(z) \geq 0$ , and

$$\lambda(\theta) = \theta \sin \theta + \cos \theta \log \cos \theta, \quad \left( |\theta| \leq \frac{\pi}{2} \right),$$

then the curve  $\Sigma$  is given by  $\sigma = \log \cos t$ ,  $\left( |t| \leq \frac{\pi}{2} \right)$ ,

and is a singular line for the function  $H(s)$ .

4. Appealing to another result of the paper<sup>6</sup> already referred to, we can deduce the following theorem:—

**THEOREM II.** *If in theorem I,  $\alpha_1 > \frac{\pi}{2}$ , and  $f(z)$  is such that  $f[\lambda(n)] = 0$  for  $n = p, p+1, p+2, \dots$  then  $f(z) \equiv 0$ .*



For  $H(s)$  is now identically zero, and so  $J(s)$  must be an integral function of  $s$ . But, since  $a_1 > \frac{\pi}{2}$ ,  $J(s)$  is identically zero, and so  $f(z) \equiv 0$ .

### References

- <sup>1</sup> Hardy: *The General Theory of Dirichlet's Series*, Theorem 10.
- <sup>2</sup> Hardy: *Orders of Infinity*, p. 17.
- <sup>3</sup> Hardy: *Orders of Infinity*, p. 34.
- <sup>4</sup> Whittaker and Watson: *Modern Analysis*, Third Edition, p. 71.
- <sup>5</sup> Srivastava and Jain: *Bul. Acad., Sci. U. P.* 2, 60, 1932.
- <sup>6</sup> The same reference as 5, p. 63.

# ON THE ABSORPTION SPECTRA OF PbO AND PbS

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The object of the present experiments is to extend Frank's work on the absorption spectra of saturated alkali halides to a new group of saturated compounds, *viz.*, the oxides and sulphides of di-valent and poly-valent atoms. Very little work has been reported on the absorption spectra of compounds of this type. Experiments on  $\text{SO}_3$  by A. K. Dutta,<sup>1</sup> on  $\text{N}_2\text{O}_5$ ,  $\text{MoO}_3$ , and  $\text{TeO}_3$ , by A. K. Dutta, and P. K. Sen Gupta,<sup>2</sup> and  $\text{CdO}$  and  $\text{ZnO}$  by P. K. Sen Gupta<sup>3</sup> have already been reported. In the present paper experiments on the absorption spectra of  $\text{PbO}$  and  $\text{PbS}$  are reported.

## EXPERIMENTAL

Owing to the high melting points of these substances, *viz.*, about  $875^\circ\text{C}$ . for  $\text{PbO}$  and  $1100^\circ\text{C}$  for  $\text{PbS}$  the vacuum graphite furnace of this laboratory was used for heating these substances. At first the salts were put on a silica tube; but on heating it was found that lead glass was easily formed. So the salts were put on asbestos pieces rolled into tubular form which was then inserted in the graphite tube. Asbestos was found not to give any appreciable vapour pressure at the temperatures required. In order to prevent the dissociation of  $\text{PbO}$  the furnace was evacuated and oxygen at a pressure of 50 cm. was filled in. In the case of  $\text{PbS}$  the spectra were taken first with the furnace completely evacuated and then with nitrogen at a pressure of 50 cm. This was necessary to prevent the oxidation of sulphur evolved by the dissociation of  $\text{PbS}$ .

The source of continuous light was a hydrogen tube run by a 2 KW Transformer. Photographs were taken by an  $E_3$  quartz spectrograph; the time of exposure being from 2—5 minutes. Copper arc was used for comparison.

## RESULTS

### PbO [yellow]

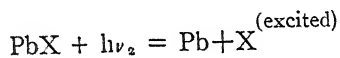
At lower temperatures the 2833A line of lead was prominent indicating that there was a partial decomposition of the oxide. As the temperature was further increased this line broadened out and at about 950°C merged into continuous absorption. This begins with a few bands at 3000 A.U. There is a retransmission followed by a second cut at 2240 A.U.

### PbS

The absorption spectra shows a continuous absorption also beginning with a few bands at 3500 A.U. A retransmission in this case also follows with a second cut at 2450 A.U.

## CALCULATION

The photo-chemical reaction may be represented by the following equation; since continuous absorption according to Frak-Condon principle indicates the breaking up of the molecule into constituent atoms,  $PbX + h\nu_1 = Pb + X$ : where X stands for O or S atom. The dissociation may, however, also occur as



where  $h\nu_2 - h\nu_1$  is the excitation energy of X.

From Born cycle we get

$$R = h\nu = Q + \frac{1}{2} D_{X_2} + L_{Pb} - L_{PbX} + L_X \dots (1)$$

Where  $Q$  = the latent heat of formation of  $[Pb X]$  from  $[Pb] + \frac{1}{2} [X_2]$

$D_{X_2}$  = the heat of dissociation of  $X_2$

$L_{Pb}$  = the latent heat of Pb.

$L_{PbX}$  = " " " " PbX.

$L_X$  = " " " " X.

The thermo-chemical data involved in this equation were taken from the tables of Landölt and Börnstein. They are tabulated below together with the various values for R.

TABLE 1. PbS

Limit of Absorption.	R K cal	Q K cal	DS.* K cal	L <sub>Pb</sub> K cal	L <sub>S</sub> K cal	L <sub>PbS</sub> K cal
- 3500A	80.2	22	56.7	44.5	15	55.6
2450A	116.7					

The value 55.6 K cal. for L<sub>PbS</sub> was obtained with the help of the relation  $L = R \frac{T_1 T_2}{T_2 - T_1} \log_e \frac{P_2}{P_1}$  from the following vapour pressure data given by Schneek and Albers\*

t	p	t	p
850°C	2 mm	968°C	10.5 mm
917°C	4 mm	995°C	17 mm

The first reading for vapour pressure gives widely different values for the latent heat when combined with the others hence it was discarded. The other three gave consistent values; the mean being 55.6 K cal.

TABLE 2. PbO

Limit of Absorption	R K cal	Q K cal	$\frac{1}{2} \text{DO}_2$ K cal	L <sub>Pb</sub> K cal	L <sub>PbO</sub> K cal
3000A	95.8	52.7	64	44.5	?
2240A	127.7				

No vapour pressure data for PbO are available. Hence its latent heat cannot be calculated.

PbS Applying the equation (1) to PbS we get

$$80.2 = 22 + 56.7 + 15 + 44.5 - L_{\text{PbS}}$$

or  $L_{\text{PbS}} = 58.0 \text{ K cal.}$

$$*S_2 + 90 = 2S$$

$$4S_6 - 29 = 3S_3$$

$$S_6 + 64 = 3S_2$$

$$S_3 + 455 = 8S.$$

These data are taken from the tables of Landolt and Bornstein.



This agrees well with the value obtained from vapour pressure data, *viz.*, 55.6 K cal. The second cut is 1.59 volts higher than the first and may correspond to the excitation energy of sulphur from the state  $3P_0$  to  $1D_2$ . This has been found by Christie and Norde<sup>5</sup> to be 1.6 volts.

**PbO** Using the relation (1) we get  $95.3 = 52.7 + 64 + 44.5 - L_{\text{PbO}} - L_{\text{PbO}}$  comes out to be 66 K cal. This is a probable value since the latent heats of oxides are generally higher than those of the corresponding sulphides.

The difference between the first and the second cut is 32.4 K cal; which may correspond to the energy required to excite the oxygen atom from  $3P_0$  state to  $1D_2$  state; which is 1.9 volts.

### CONCLUSION

In the normal state the molecule may be assumed to be  $\text{Pb}^{++} \text{X}^{--}$  extending Frank's view as suggested by P. K. Sen Gupta. O has the same configuration as Neon and S as Argon. The ground state of  $\text{Pb}^{++}$ , which has the same configuration of electrons as Mercury is  $1S_0$ . The salts should be both diamagnetic.

Corresponding to the fact that the molecules are quite stable the potential energy curve will have a deep minima. It will be of the type L represented in the figure 1.

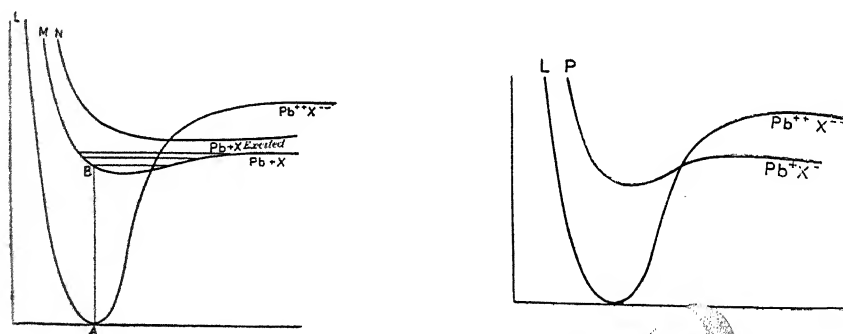


Fig. 1

By absorption of light the molecule will pass along the next curve M without change of nuclear distance in general, as is in accordance with Frank-Condon principle. This will thus result in the dissociation of the molecule into  $\text{Pb} + \text{X}$ . The presence of a few absorption bands may be due to the continuous absorption may be due to the fact that the molecule may have a flat minima. The greatest value of the potential energy is along AB when the molecule passes through the minimum of the normal state. The next higher curve is the excited state of the molecule so that X is excited to some upper state P (Fig. 2) of the molecule  $\text{Pb}^{++} \text{X}^{--}$  with

L to P will give rise to band absorption. Attempts to search for these probable bands are being made.

On the assumption that the normal state is  $\text{Pb}^{++} \text{X}^{--}$  the binding must obviously be electrostatic. A study of the lower half of Born cycle seems to lend some support to this view.

Here  $E$  = the lattice energy of the crystals.

$I_1$  = the ionisation potential of Pb

$I_2$  = the ionisation potential of  $\text{Pb}^+$ .

$I_e$  = two electron affinity of X

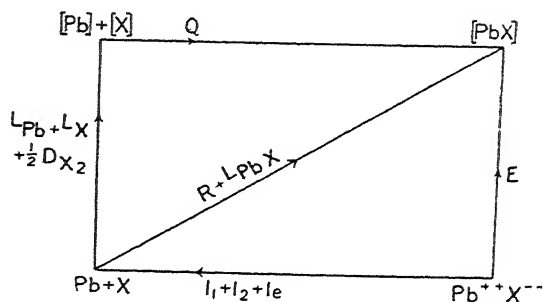


Fig. 3

All other symbols have already been defined.

From the lower half of Born cycle we get

$$E - (I_1 + I_2 + I_e) = R + L_{\text{PbX}} \quad (2)$$

#### CALCULATION OF E

The value of  $E$  for PbS has been calculated by Born and Gerlach<sup>11</sup> with the help of the formula

$$E = K \cdot \frac{n-1}{n} \sqrt[3]{\rho_{\text{IM}}}$$

where  $K$  = a constant involving Madelungs constant

$n$  = repulsion exponent

$\rho$  = density

$M$  = molecular weight.

The repulsion exponent  $n$  is given as  $n = 1 + 8.00 \times 10^{-11} \cdot \frac{1}{\chi} \left( \frac{M}{P} \right)^{\frac{4}{3}}$ ,  $\chi$  being the compressibility.

The lattice energy of PbS whose crystals are cubical of Sodium chloride type has been found by these authors as 636K cal.

The lattice energy of PbO unfortunately cannot be calculated; since there is no data for its compressibility.

The  $^3\text{P}_0$  state of Pb = 59810<sup>7</sup>

$^2\text{P}_{\frac{1}{2}}$  state of  $\text{Pb}^+$  = 121256<sup>6</sup>

Hence  $I_1 + I_2 = 517\text{K cal.}$

The double electron affinity of S has been calculated by Samuel and Lorentz<sup>1</sup> and they find it as  $-30$  K cal.

The left hand side of equation (2) thus gives

$$636 - (517 - 30) = 149 \text{ K cal}$$

while the right hand side gives  $R + L_{\text{PbS}} = 137.1 \text{ K cal.}$

The agreement is fairly good considering the uncertainties involved in the calculation of the lattice energy and the electron affinity of PS.

My thanks are due to Prof. M. N. Saha, F.R.S., for his kind interest in this work.

### SUMMARY

1. The absorption spectra of PbO and PbS has been studied. There is a continuous absorption in both the cases showing that the photo-chemical reaction results in the dissociation of the molecules into two free atoms.

2. Retransmission in each case is interpreted as due to the excitation of sulphur or oxygen atom from the normal state  $^3P_0$  to the metastable state  $^1D_2$ .

3. The latent heat of PbS is found to be  $58.0$  K cal; agreeing well with the value obtained from vapour pressure data, viz.,  $55.6$  K cal. The latent heat of PbO is found to be  $66$  K cal.

4. A study of the Born cycle lends some support to the view that in the normal state the molecules are  $\text{Pb}^{++}\text{X}^{--}$ .

### References.

- <sup>1</sup> Dutta, A. K., *Proc. Roy. Soc. A*, 137, 336 (1932).
- <sup>2</sup> Dutta, A. K. and Sen Gupta, P. K., *Proc. Roy. Soc.*, 139, 397 (1933).
- <sup>3</sup> Sen Gupta, P. K., *Bul. Acad. Sci. U. P.* 2, 245 (1933).
- <sup>4</sup> Schenk and Albers, *Landolt and Börnstein Tables*.
- <sup>5</sup> Christie and Norde, *Phys. Rev.* 37, 903 (1930).
- <sup>6</sup> Born and Gerlach, *Zs. F. Phys.* 5, 441 (1921).
- <sup>7</sup> Grotrian and Geisler, *Zs. F. Phys.* 39, 377 (1926).
- <sup>8</sup> Geisler, *Zs. F. Phys.* 42, 265 (1927).
- <sup>9</sup> Samuel and Lorentz, *Zs. F. Phys.* 59, 59 (1930).

# THE ABSORPTION SPECTRA OF THE VAPOURS OF THE LOWER CHLORIDES OF ELEMENTS OF THE FIFTH GROUP OF PERIODIC TABLE

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The object of the present paper is to report experiments on the absorption spectra of the trivalent halides of elements of the fifth group, *viz.*, P, As, Sb and Bi. As is well known they form pentahalides as well as trihalides. All available information about them is collected in the following table.

TABLE I.

Compound		Trichloride	Tribromide	Triiodide	Pentachloride	Pentabromide
Element						
Phosphorus	...	$\text{PCl}_3$ Colourless liquid b.p. $76^\circ\text{C}$ m.p. $-112^\circ\text{C}$ .	$\text{PBr}_3$ Colourless liquid b.p. $170.8^\circ\text{C}$ m.p. $-40^\circ\text{C}$	$\text{PI}_3$ Dark red crystals m.p. $61^\circ\text{C}$ .	$\text{PCl}_5$ White crystalline solid, sublimes m.p. $148^\circ\text{C}$ b.p. $162^\circ\text{C}$	$\text{PBr}_5$ Yellow crystalline solid, decomposes on heating b.p. $106^\circ\text{C}$
Arsenic	...	$\text{AsCl}_3$ Colourless oily liquid b.p. $130.2^\circ\text{C}$ m.p. $-13^\circ\text{C}$	$\text{AsBr}_3$ Colourless crystalline solid m.p. $31^\circ\text{C}$ b.p. $221^\circ\text{C}$	$\text{AsI}_3$ Red hexagonal crystals m.p. $146^\circ\text{C}$	$\text{AsCl}_5$ Decomposes into $\text{AsCl}_3$ and $\text{Cl}_2$ above $-25^\circ\text{C}$	...
Antimony	...	$\text{SbCl}_3$ White crystalline solid, m.p. $73.2^\circ\text{C}$ b.p. $223.5^\circ\text{C}$	$\text{SbBr}_3$ White deliquescent needles m.p. $73^\circ\text{C}$ b.p. $280^\circ\text{C}$	$\text{SbI}_3$ Melting point of the stable form is $171^\circ\text{C}$	$\text{SbCl}_5$ Yellow mobile liquid b.p. $140^\circ\text{C}$	$\text{SbBr}_5$

TABLE I—(continued)

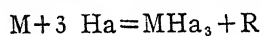
Compound		Trichloride	Tribromide	Triiodide	Pentachloride	Pentabromide
Element						
		$\text{BiCl}_3$	$\text{BiBr}_3$	$\text{BiI}_3$	$\text{BiCl}_5$	$\text{BiBr}_5$
Bismuth	...	White crystalline solid m.p. $227^\circ\text{C}$ b.p. $447^\circ\text{C}$	Golden yellow crystals.	Black powder.		

The question now arises as to whether the trihalides or the pentahalides can be regarded as saturated compounds. Let us first take as an illustrative example, the chlorides of phosphorus.

$\text{PCl}_3$  is diamagnetic, we may suppose that it has the constitution  $\text{P}^{+++}\text{Cl}^{-}$ . Each  $\text{Cl}^{-}$  ion is diamagnetic, and  $\text{P}^{+++}$  has the constitution  $1s^2 2s^2 2p^3 3s^2$ ; hence  $\text{P}^{+++}$  is also diamagnetic.

$\text{PCl}_5$  is also diamagnetic. We may suppose that it has the constitution  $\text{P}^{+5}\text{Cl}^{-}$ .  $\text{P}^{+5}$  has the inert gas constitution  $1s^2 2s^2 2p^6$ . If the above hypothesis be true, both  $\text{PCl}_3$  and  $\text{PCl}_5$  should show continuous absorption, as in the case of saturated halides of different valency. Chlorides of other elements in this group of periodic table will also behave likewise.

The calculation of the atomic heat of formation of  $\text{MHa}_3$  out of one M atom and 3 Ha atoms as expressed by the relation



is a matter of some difficulty; for usually the elements of this class, *viz.*, P, As, Sb and Bi vaporise in the polyatomic state. Preuner and Brockmüller<sup>1</sup>, in their extensive studies on the vapour pressure and vapour density of P and As find that at ordinary temperatures they vaporise as  $\text{P}_4$  and  $\text{As}_4$  which however break up into  $\text{P}_2$  and  $\text{As}_2$  as higher temperatures are reached. At still higher temperatures, the diatomic molecules may be broken up into atoms. The processes may be thus represented

$$[\text{P}] = \text{P}_4 + \lambda$$

$$\text{P}_4 = 2\text{P}_2 - \text{D}_1$$

$$\text{P}_2 = 2\text{P} - \text{D}_2$$

According to Preuner and Brockmüller<sup>1</sup>

$$\text{D}_1 = 31.5 \text{ K cal}; \text{D}_2 = 45.5 \text{ K cal} \text{ and } \lambda = 12.6 \times 124 = 15 \text{ K cal.}$$

Let  $S$  be the heat of vaporisation from the condensed state to the atomic state. Then we have

$$S = \frac{\lambda + D_1 + 2D_2}{4} = \frac{15 + 31.5 + 91}{4} = \frac{138}{4} = 34.5 \text{ K cal.}$$

Then

$$R = Q + S + \frac{3}{2} D_{\text{HCl}} - \lambda_{\text{M}_{\text{HCl}}}$$

In the case of  $\text{PCl}_3$

$$Q = 75.3 \text{ K cal.}; S = 34.5 \text{ K cal.}; \frac{3}{2} D_{\text{HCl}} = 86 \text{ K cal.}; \lambda_{\text{M}_{\text{HCl}}} = 9.3 \text{ K cal.}$$

$$R = 186.5 \text{ K cal.}$$

The Table II has been compiled as shown below.

As regards Sb and Bi the dissociation has not been well studied. But probably they also vaporise as  $\text{Sb}_4$  and  $\text{Bi}_4$ . For Bismuth there is some indirect evidence. The absorption spectrum of Bi vapour has been studied by Barratt and Bonar<sup>2</sup> and has been found to consist of two indistinct systems of bands. One of them has been attributed to  $\text{Bi}_2$  molecule and from the convergence limit of this band the heat of dissociation of  $\text{Bi}_2$  molecule has been calculated amounting to 18.5 K cal. The other band system has been attributed to a molecule of greater complexity, probably  $\text{Bi}_3$ ; but unfortunately, it has not been analysed and we do not know the heat of dissociation of  $\text{Bi}_3$ .

Since all these molecules belong to the same group of periodic table we expect that the nature of their binding forces will be similar. As we know these values for a few of them we can get them for the others by interpolation. The interpolated values give at least the region where the correct values should lie. We take these values for the calculation of  $R$  in respective cases. The extrapolated values and their justification is to be seen from Tables 1, 2 and 3.

TABLE I

Reaction $\text{M}_2 \rightarrow 2\text{M}$	Heat of dissociation ( $D_2$ )	Difference
$\text{P}_2 \rightarrow 2\text{P}$	45.5 K cal	
$\text{As}_2 \rightarrow 2\text{As}$	38 K cal	7.5 K cal
$\text{Sb}_2 \rightarrow 2\text{Sb}$	(28) K cal	(10) K cal
$\text{Bi}_2 \rightarrow 2\text{Bi}$	18.5 K cal	(9.5) K cal

TABLE 2

Reaction $M_4 \rightarrow 2 M_2$	Heat of dissociation ( $D_1$ )	Difference
$P_4 \rightarrow 2 P_2$	31.5 K cal	6.5 K cal
$As_4 \rightarrow 2 As_2$	25 K cal	(9) K cal
$Sb_4 \rightarrow 2 Sb_2$	(16) K cal	(8.5) K cal
$Bi_4 \rightarrow 2 Bi_2$	(7.5) K cal	

TABLE 3

Substance	Heat of dissociation ( $D_2$ )	Heat of dissociation ( $D_1$ )	Difference
Phosphorus	45.5 K cal	31.5 K cal	14 K cal
Arsenic	38 K cal	25 K cal	13 K cal
Antimony	(28) K cal	(16) K cal	(12) K cal
Bismuth	18.5 K cal	(7.5) K cal	(11) K cal

TABLE II

N.B.—The values enclosed in brackets ( ) are uncertain because of the above interpolation.

Substance	Long wavelength limit $\nu_m$ in $^{\circ}A$	$Q_m = \frac{Nh\nu_m}{J}$ in K cal	Heat of reaction required to convert solid element into monatomic vapour in K cal per gm. atom "L"	Heat of formation of the salt in K cal per mole "Q"	Heat of vaporisation of the salt in K cal per mole " $\lambda$ "	$\frac{R}{3}$ in K cal	$Q_m - \frac{R}{3}$ in K cal
Phosphorus Trichloride	2957	96.2	34.5	75.3	9.25	62.5	33.7
Arsenic Trichloride	5466	82.1	26.5	71.4	6.7	59.4	22.6
Antimony Trichloride	3256	87.3	(27.6)	91.4	11.05	(65.0)	(22.3)
Bismuth Trichloride	3656	77.2	(24.2)	90.6	18.3	(61.2)	(16.0)

## EXPERIMENTAL PROCEDURE

Of the four substances examined two were liquids and the other two were solids whose boiling points were less than  $500^{\circ}\text{C}$ . Two kinds of absorption vessels were used for producing the vapour and studying the absorption. For the solids (*i.e.*,  $\text{BiCl}_3$  and  $\text{SbCl}_3$ ) a pyrex glass furnace of the design shown in the diagram below was used (Fig. 1).

A is a pyrex glass tube of about one inch in diameter. BB are water-jackets employed for preventing the hot vapour from coming from the inside of the tube and depositing on the quartz windows C. The substances could be introduced in through the side tube D which also served for the connection to the vacuum pump. The furnace was heated by winding manganin wire round it and passing high electric current through it.

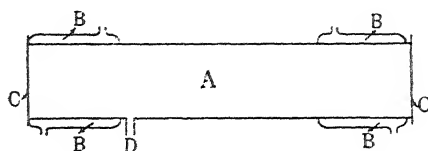


Fig. 1

For the liquid the furnace took the form of a long tube of glass of about an inch in diameter closed at its ends by quartz windows. Two side tubes provided with stop-cocks were attached to it. One of these tubes served for the connection to the vacuum pump. The other ended in a glass bulb of about 50c.cs. in capacity which was provided with a second stop cock for connecting the bulb to the atmosphere (Fig. 2).

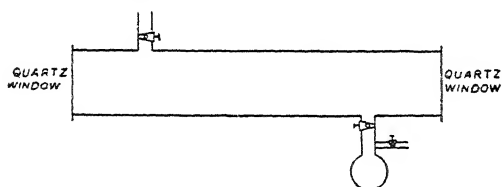


Fig. 2

The liquid under investigation, could be introduced into the side bulb. The vapour pressure in the long tube was varied by placing the bulb in baths of varying temperatures.

The vapour pressures could be measured in the first case by noting the temperature to which the furnace had been raised and taking from the tables (given in Mellor's 'Comprehensive Treatise in Inorganic and Physical Chemistry') the value of the saturation vapour pressures of the substance at that temperature. The temperature of the furnace was known by calibrating it in terms of the heating current when it had come to a steady state of temperature. In practice the steady state was brought about by heating it for at least ten minutes with constant current. Each photograph was taken only when the furnace had been heated by a definite electric current for about 30 minutes. In case of liquids the vapour pressure was measured by introducing a manometer on the pump side of the furnace.



The source for the continuous absorption was the Hydrogen tube run by a high current transformer at a current density of 100 M.A. The photographic plates used were Ilford Process plates.

#### DISCUSSION

We find (cf. Table II) that the energy of optical dissociation is in each of the compounds greater than the one-third of thermochemical value but less than two-thirds of it. Thus although one chlorine atom is shaken off from the molecule by the incident radiation, the energy expended is not strictly equal to that present in a single bond of the molecule.

Although a definite interpretation of these results cannot be offered unless more data accumulates, yet a few observations may well be made.

When a quantum of light, of energy  $h\nu$  ( $\nu$  being the long wavelength limit of the absorption spectrum) comes into contact with the molecule, it is absorbed by the latter. On its absorption it excites the electronic structure of the molecule which now corresponds to the state represented by a dipole  $MCl_2 - Cl^*$ . Here one electron passes over from the chlorine ion to the remaining portion of the molecule and we get dissociation product of the molecule, *i.e.*, a chlorine atom and a dichloride of the metal. It is found that the energy corresponding to the long wavelength limit of absorption is greater than the value for  $\frac{R}{3}$  in the case of all these chlorides. Franck in a recent note to the 'Naturwissenschaft' suggests that this difference is due to the vibrational energy given to the dichloride, which is obtained on photo-dissociation. The difference found in the present case is of the order of twenty to thirty K cal. This order of magnitude is too big to be due to vibrational changes only. It seems probable that this extra energy is involved in the electronic structure of the dichloride formed. The dichlorides are found as definite compounds, but unfortunately there does not exist any data about their heats of formation. This assumption is plausible as the electronic structure of the dichloride in the undissociated trichloride is not the same as that associated with the free dichloride. To bring about this change in the electronic structure some energy is obviously required. It manifests itself in the difference between  $\frac{R}{3}$  and  $Q_m$ .

The results may also be explained from a new hypothesis put forward by M. S. Desai of this laboratory (the hypothesis is yet unpublished for want of confirmatory experimental support). Desai considers the strength of each bond separately, making an assumption that if the fully developed compound be  $MX_n$  the unsaturated state  $MX_m$  ( $m < n$ ) will have a binding strength  $\frac{m}{n}$  times the fully developed compound. He calculates the strength

of each bond separately and adding them together equates it to the total thermo-chemical energy  $R$  and obtains the fraction which should give the beginning of absorption. He gets, for

$MX_4$	the limit of absorption to correspond to	$\frac{2}{3} R$
$MX_3$	" " " "	$\frac{1}{3} R$
$MX_2$	" " " "	$\frac{2}{3} R$
$MX$	" " " "	$R$

In the present case the absorption limit should be obtained by  $\frac{R}{2}$  and not  $\frac{R}{3}$  as given by Datta and Saha<sup>3</sup>. The agreement is seen from the following figures:—

TABLE 4

Substance	$Q_m$ (observed) K cal s	$\frac{R}{2}$ K cal s	$\frac{R}{3}$ K cal s
Phosphorous Trichloride ...	96.2	94.0	62.1
Arsenic Trichloride ...	82.1	86.5	57.4
Antimony Trichloride ...	87.3	(97.5)	(65.0)
Bismuth Trichloride ...	77.2	(92.2)	(61.2)

The agreement is tolerable in the case of  $PCl_3$  and  $AsCl_3$ . In the case of other two compounds nothing can be said either for or against Desai's hypothesis, as the heats of dissociation of  $Bi$ ,  $Sb$ , and  $Sb_2$  are not known. The value of  $R$  calculated by the help of the extrapolated values for those quantities cannot be accurate.

## ACKNOWLEDGMENTS

I have much pleasure in acknowledging my indebtedness to Prof. M. N. Saha, without whose extreme kindness on the author, this paper would not have seen the light of the day. My heartiest thanks are due to Prof. A. T. Mukerji of Patna for his having kindly lent me the use of the photomicrometer

and to Mr. Bholā Nath Ghosh research scholar Patna for rendering me material help during my stay there.

*References*

- <sup>1</sup> G. Preuner and I. Brockmüller, *Zeit. f. Physik. Chem.*, 8, 129, 1913.
- <sup>2</sup> S. Barratt and A. R. Bonar, *Phil. Mag.* 9, 519, 1930.
- <sup>3</sup> A. K. Datta and M. N. Saha, *Bull. Acad. Sc. U. P.*, 1, 29, 1931-32.

# A NOTE ON THE VAPOUR PRESSURE OF ZINC BROMIDE

By M. S. DESAI

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Communicated by Prof. M. N. Saha.

Received April 3, 1933.

The purpose of this note is to correct a mistake that has been discovered after the publication of the paper on the same subject by the author,<sup>1</sup> in the last issue of the Bulletin.

In the said paper it is assumed that,

$$\begin{aligned} C - C_p &= 3R - \frac{5}{2}R \\ &= R/2 \end{aligned}$$

where  $C$  is the specific heat of the solid and  $C_p$  that of the vapour at constant pressure, but this is only true in the case of monatomic vapour. In the case of  $\text{Zn Br}_2$ , where the molecule is tri-atomic it requires modification.

$$C = 3R; \text{ and } C_p \text{ is given by } C_p - C_v = R$$

and

$$\begin{aligned} C_v &= C_{\text{rot}} + C_{\text{trans}} + C_{\text{osc.}} \\ &= \frac{3}{2}R + \frac{3}{2}R \quad (\text{neglecting the oscillation at low temp.}) \\ &= 3R \end{aligned}$$

$$\text{therefore } C_p = 3R + R = 4R$$

$$\begin{aligned} \text{and } C - C_p &= 3R - 4R \\ &= -R \end{aligned}$$

Thus Clausius' equation  $\lambda = RT^2 \frac{d \ln P}{dT}$  becomes,

$$\ln P = -\frac{\lambda_0}{RT} - \frac{C - C_p}{R} \log T + k$$

$$\text{or } \log_{10} P = \frac{-\lambda_0}{2.3RT} + \log_{10} T + k'$$

This is of the form,

$$\log P = -\frac{A}{T} + \log T + B$$

Evaluating the constants A and B from the experimental data we have,

$$\left. \begin{array}{l} A = 5639.781 \\ \text{and } B = 9.066 \end{array} \right\} \dots \dots \dots (A)$$

Thus the vapour pressure equation for  $\text{Zn Br}_2$ , between  $300^\circ$  to  $400^\circ\text{C}$  becomes,

$$\log_{10} P = \frac{-5639.781}{T} + \log_{10} T + 9.066$$

And the latent heat of evaporation

$$\begin{aligned} \lambda_0 &= -4.554 A \\ &= 4.554 \times 5639.781 \\ &= 25.68 \text{ k cals} \dots \dots \dots (B) \\ &\text{instead of } 27.60 \text{ k cals as calculated before.} \end{aligned}$$

My thanks are due to Mr. B. N. Srivastava of our Department for pointing out the mistake.

*Reference.*

<sup>1</sup>M. S. Desai, *Bul. Acad. Sci.*, U. P., Allahabad, 2, 124, 1933.

ON THE TREMATODE PARASITES OF A RANGOON SILUROID  
FISH *CLARIAS BATRACHUS* (LINNAEUS 1785)

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Communicated by Dr. H. R. Mehra

Received March 26, 1933.

In course of an investigation on the parasites of the common food fish of Rangoon, there were obtained from thirty *Clarias batrachus* (Linnæus 1785) a large number of trematodes which fall into three distinct groups. Two of these are members of the family Lepodermatidae Odhner 1911—one representing a new species and the other a new genus: the third is markedly different from the others and constitutes a new genus of a new sub-family belonging to the family *Acanthostomidae* Poche 1926.

***Astiotrema Spinosa* n. sp.**

These trematodes do not appear to be common: out of the thirty fish examined only three were found to be infected, with 35, 9, and 1 parasites respectively. In hosts one and three the parasites were obtained from the intestine and in host two from the posterior part of the stomach. Body when

contracted, pegshaped, with a much broader anterior end: when elongated margins more or less parallel, with rounded anterior and posterior ends. In the extended condition the anterior end is greatly narrower ending in a blunt point. Body musculature more strongly developed between suckers, so that, when the animal contracts, the two suckers lie very close to each other. Cuticle covered with sharp pointed spines arranged in transverse rows, the spines of each row alternating with those of the preceding and succeeding rows. Spines become progressively denser on the surface from the posterior to the anterior part of the body. Body length 1.3\*—3, maximum breadth 0.4—0.58, with a much smaller variation than in the length. Suckers powerful. Ventral sucker in first third of the body, circular in outline, approximately 0.185 in diameter. Oral sucker, slightly smaller than ventral, 0.18 × 0.16. Genital opening closely anterior to ventral sucker. Small prepharynx present. Pharynx spherical, 0.08 in diameter in extended specimen. Oesophagus moderately long, from 1—2½ times length of pharynx. In rare cases, *i.e.*, in fully extended condition its length extends to four times the length of pharynx. Intestinal bifurcation between the two suckers, its position changing with the degree of expansion. Intestinal caeca more or less straight, terminating a little anteriorly to posterior end of body.

Testes rounded, large, usually equal, 0.16—0.28 in diameter, situated obliquely one behind the other in posterior half of body just behind the middle, occupying most of the space between the caeca. In a few preserved and mounted specimens, however, the testes were found to be elliptical and broader than long. Cirrus sac large, extending much behind the ventral sucker as far as ovary, 0.23—0.44 in length, posteriorly saccular, almost entirely filled by the large vesicula seminalis and continuing anteriorly into a tubular structure. Parsprostatica tubular and long separated from vesicula seminalis by constriction. Ductus ejaculatorius small, continuing into a small cirrus. Male opening shallow, genital atrium to the right of the female opening.

Ovary spherical, 0.13—0.17 in diameter, approximately in median line, much posterior to ventral sucker except in contracted specimens. Receptaculum seminis large, pearshaped and elongated transversely, joining oviduct near ovary. Coils of uterus rarely overlapping gut diverticula except in the part posterior to testes. Vitellaria composed of numerous follicles, scattered without any definite arrangement along lateral margins of body overlapping intestinal caeca from posterior margin of ventral sucker to anterior margin of posterior testis or sometimes to posterior margin of the latter, in which case the distance between the two testes was considerably less than normal and that from the posterior testis to posterior end of the body correspondingly

\* All measurements are in millimetres.

increased, indicating an anterior displacement of that testis. The follicles greatly extend inwards to the median line in the region between ovary and anterior testis. Eggs numerous, with a yellowish-brown shell,  $0.027 \times 0.0114$ .

The present species has a small protrusible cirrus and a large cirrus sac extending much behind the ventral sucker which is characteristic of the genus *Astiotrema*. It thus agrees with the definitions given by various investigators (Looss, Odhner, and Mehra). The Burmese form differs from *Astiotrema loossii* Mehra 1930 in the relative size of its suckers, in the absence of lobed margins to the testes and ovary, and in the extent of vitellaria and intestinal caeca. From *A. impletum* Looss 1899, *A. monticellii* Stossich 1904 and *A. elongatum* Mehra 1930 it is separated by a difference in the position of the intestinal fork and by the extent of intestinal caeca and vitellaria. It is closer to *A. reniferum* Looss 1899 but differs from that species in the extent of the intestinal caeca and vitellaria. These differences necessitate the creation of a new species, for which the name *Astiotrema spinosa* is proposed. This is the second species of the genus *Astiotrema* obtained from fish, the first *Astiotrema impletum* Looss 1899, being recorded from *Tetradon fahaka*.

#### ***Ganada clariae* n. gen., n. sp.**

This species is common: out of the fish examined nineteen were infected, the number of parasites from each fish ranging from 1-63, with an average of 10-15. Body, when contracted, oval, with much broader anterior end: when extended, cylindrical, with rounded ends. Length 1.53-2.8 maximum breadth 0.3-0.4. Cuticle with small spines, progressively denser from posterior to anterior part of body. Suckers powerful, ventral slightly larger than oral,  $0.133-0.18$  and  $0.114-0.164$  in diameter respectively. Genital pore immediately in front of ventral sucker to the left of the median line and some distance behind intestinal bifurcation. Prepharynx short,  $0.04-0.046$  in length. Pharynx  $0.08-0.14$  in diameter. Oesophagus  $0.027-0.061$  in length. Intestinal caeca wide, comparatively broad anteriorly and extending almost to posterior end of body.

Testes median, in the third quarter of the body, the distance separating the two differing in different specimens. Posterior testis slightly bigger than anterior, approximately  $0.152-0.21$  and  $0.137-0.18$  in diameter respectively. Genital atrium small,  $0.02-0.04$  in diameter. Cirrus sac somewhat tubular, slightly semilunar,  $0.234-0.4$  in length, dorsal to ventral sucker extending slightly posteriorly to it. Vesicula seminalis divided into internal and external portions, the latter lying close to ovary. Pars prostatica small, near anterior margin of ventral sucker. Cirrus small. Male opening to the right side of the female.

Ovary approximately spherical,  $0.114-0.162$  in diameter, lying in middle third of body. Vitellaria composed of numerous, closely crowded big follicles



along lateral margins of body, more densely crowded in post-testicular region than in pre-testicular. Uterus much coiled, the loops running transversely and passing between testes. Eggs numerous, thin-shelled, approximately  $0.018 \times 0.012$ . Excretory bladder Y-shaped and typical of the family Lepodermatidae Looss 1899.

The parasite agrees with the characters of the subfamily Lepodermatinae Looss 1899, but differs from all the genera included in it in the presence of an external vesicula seminalis. In the absence of a receptaculum seminis it resembles *Lepoderma* Looss 1899 and *Haplometra* Looss 1899 but differs from the other genera. The tandem position of the testes and the position of the cirrus sac further separate it from *Lepoderma* Looss 1899. From *Haplometra* Looss 1899 it differs in the rudimentary nature of its oesophagus and the pronounced coiling of the uterus in addition to the presence of its characteristic external vesicula seminalis. By its possession of the latter organ it exhibits a certain affinity with the genus *Leptophallus* Luhe 1909, but this similarity does not extend to other characters. A new genus is therefore created for the reception of these parasites, for which is proposed the name *Ganada* with *Ganada clariae* as the type genus and species.

#### Diagnosis of the genus *Ganada*. n. gen.

Lepodermatidae: Lepodermatinae. Cuticle with spines. Suckers almost spherical, unequal, Prepharynx and pharynx present. Oesophagus very short. Genital pore left of the median line, anterior to ventral sucker. Intestinal caeca extending to hinder part of body. Testes post-ovarian and median, the posterior larger than the anterior. Ovary smaller than testes. Cirrus sac semilunar and median, extending dorsally over ventral sucker. Vesicula seminalis divided into internal and external vesiculæ seminales. Receptaculum seminis absent. Vitellaria with closely packed follicles, extending from behind ventral sucker to posterior end of body. Uterus much coiled, nearly reaching posterior end of body and containing numerous eggs. Excretory bladder Y-shaped and typical of the sub-family.

#### \**Masenia collata* n. gen., n. sp.

On examination of thirty fish, eighteen were found infected with these parasites, in the intestine or posterior part of the stomach, the number in each case ranging from 1—390.

\* From *masen*, the Chingpo for spine or thorn.

Body club-shaped, 0.7—1 in. length with a maximum breadth of 0.24—0.36. Cuticle for anterior three-fourths of body armed with small sharp spines which become progressively denser from the posterior to the anterior part of the body. In addition to body spines, approximately 53 oral spines are present around the oral sucker, arranged in two regular rows—one above the other. Suckers very powerful. Oral larger than ventral, 0.12—0.17 × 0.11—0.15, funnel-shaped, extending for a considerable distance within the body. Ventral sucker circular, 0.1—0.13 in diameter, lying in anterior half of body. Pharynx short, approximately 0.02—0.045 in length. Pharynx 0.04—0.54 × 0.025—0.042. Oesophagus small, dividing immediately anteriorly to ventral sucker into two wide caeca which, maintaining approximately the same width throughout, extend to the level of the posterior testis.

Testes approximately equal and spherical, 0.07—0.1 in diameter, lying close to each other in middle third of body. Genital atrium in form of a shallow depression close behind oral sucker. Cirrus sac large, of greater length than half the length of the body, divisible into two distinct portions, a broad basal part and a long narrow coiled tubular structure, the latter fitting into the neck of the former. Vesicula seminalis mostly within saccular part of cirrus sac, divided by a constriction into a smaller proximal and a large distal portion, and extending for a short distance in its tubular part. Pars prostatica and ductus ejaculatorius enclosed within narrow tubular part of cirrus sac. (In Fig. 5 not visible being hidden by gland cells). Cirrus small and in most of the fixed specimens distinctly protrudes out of the body in region of oral sucker.

Ovary immediately behind ventral sucker, approximately spherical, 0.064—0.12 in diameter, slightly bigger than testes. Receptaculum seminis large, posterior to ovary, often obscured by the large number of eggs in its vicinity. Vitellaria composed of follicles extending from anterior margin of ventral sucker to middle of posterior testis. Uterus voluminous, the convolutions occupying practically the whole body posterior to intestinal bifurcation and too closely crowded for the individual coils to be observed. Passing backwards from its origin behind the ventral sucker it forms a double sinuous course in the post-acetabular region. It then turns forward and, still with a slightly sinuous course, opens into the genital atrium immediately behind the oral sucker. Eggs numerous, oval, with a yellowish shell approximately 0.02 × 0.012. Excretory bladder broad and tubular, extending anteriorly to posterior testis; cornua not visible probably owing to the crowding of genital glands, cirrus sac and ventral sucker.

The peculiar oral spines and the funnel-shaped oral sucker of *Masenia collata* n. sp. are characteristic of some of the genera of the family *Acanthostomidae* Poche 1926 (Syn. *Acanthochoasmidae* Nicoll (1914) to which it exhibits closer relationship than to any other family. The presence of a well-developed

cirrus sac in *Masenia*, however, separates it from all the genera of the *Acanthostomidae* Poche 1926 except those included in the sub-family *Anoictostominae* Nicoll 1914. The affinities, however, are very close with *Anoictosoma planicolle* Rud. 1819) as exhibited in the following characters:—

1. Large funnel-shaped oral sucker.
2. Presence of oral spines.
3. Size of suckers.
4. Well-developed cirrus sac enclosing large vesicula seminalis.
5. Ovary close behind ventral sucker.
6. Uterine coils mainly posterior to testes.

On the other hand, the differences between them are so great that it is not possible to include *Anoictosoma planicolle* (Rud. 1819) and my species in the same genus. In most of the genera of the family *Acanthostomidae* Poche 1926, the cirrus sac has been recorded to be absent and in some cases where it is present as in *Anoictosomum* Stossich 1899 it does not extend much in front of the anterior margin of ventral sucker. *Masenia* n. gen. has a long tubular coiled cirrus sac which remarkably differs in shape and size from that of all the genera of the family *Acanthostomidae* Poche 1926. Further, the genital opening in this family lies immediately anterior to ventral sucker but in *Masenia* n. gen. it has become shifted much forward and occupies a position dorsal to oral sucker. It is therefore considered necessary to create a new genus *Masenia* with *M. collata* as the type species. Though the genus shows closer relationship with the sub-family *Anoictostominae* Nicoll 1914 than *Acanthostominae* there does not seem to be any doubt that it belongs to a new sub-family *Maseniinae* on account of two important characters, i.e., the form and size of the cirrus sac and the position of the genital pore.

Key to the sub-families of the family *Acanthostomidae* Poche 1926:—

1. Cirrus sac absent ... *Acanthostominae*.  
     Cirrus sac present ... 2.
2. Genital opening close in front of  
     ventral sucker ... *Anoictostominae* Nicoll 1914.

Genital opening far in front of ventral  
 sucker and near oral sucker... *Maseniinae* n. sub. fam.

Diagnosis of the sub-family *Maseniinae* n. sub. fam.:—

*Acanthostomidae*: Body small. Cuticle spinose. Suckers strongly developed: oral sucker funnel-shaped, much larger than ventral, with strong spines arranged in two rows. Prepharynx present. Pharynx well developed. Oesophagus of moderate length. Cirrus pouch long and coiled, divisible into a basal saccular part and a distal tubular portion nearly reaching anterior end of body. Vesicula seminalis large, its basal portion saccular. Cirrus small and unarmed. Testes post-acetabular, post-ovarian, nearly spherical. Re-

receptaculum seminis present. Vitelline glands with large follicles extending between ventral sucker and posterior testis. Uterus very voluminous. convoluted, occupying greatly hinder part of the body behind testes. Excretory bladder tubular, extending to posterior testis. Eggs numerous and oval. Parasites of siluroid fish (*Clarias batrachus*).

Type genus *Masenia* n. gen.

Type-species *M. collata* n. gen., n. sp.

#### ACKNOWLEDGMENT

The author has great pleasure to express his sincere thanks to Dr. H. R. Mehra and Professor F. J. Meggitt for their kind help and valuable suggestions. Thanks are also due to Professor D. R. Bhattacharya for kindly providing facilities to complete this work in the Zoology Department of the Allahabad University.

#### EXPLANATION OF PLATES

Fig. 1. *Astiotrema spinosa*—Ventral aspect (camera lucida drawing).

Fig. 2. *Ganada clariae* n. gen., n. sp.—Ventral aspect (camera lucida drawing).

Fig. 3. *Ganada clariae* n. gen., n. sp.—Horizontal-Longitudinal section.

Fig. 4. *Masenia collata* n. gen., n. sp.—Ventral aspect (camera lucida drawing).

Fig. 5. *Masenia collata* n. gen., n. sp.—anterior part of the body showing cirrus sac.

#### LETTERING TO FIGS. 1—5

c. cirrus, c. s. cirrus sac, e. s. v. vesicula seminalis externa, g. o. genital opening, i. c. intestinal caecum, i. s. v. vesicula seminalis interna, oes. oesophagus, o. s. oral sucker, ov. ovary, p. g. prostate gland cells, ph. pharynx, r. s. receptaculum seminis, s. g. shell gland, s. v. vesicula seminalis, t.<sub>1</sub>. anterior testis, t.<sub>2</sub>. posterior testis, ut. uterus, vit. vitellaria, v. s. ventral sucker.

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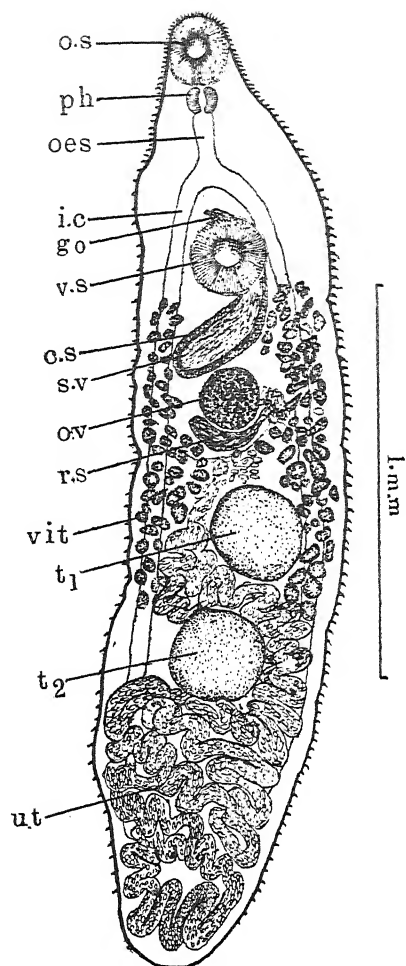


FIG 1.—*Astiotrema spinosa*, n. sp., ventral aspect.

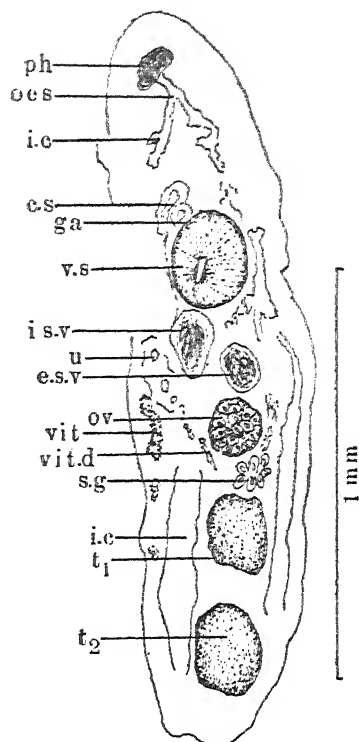


FIG 3.—*Ganada Clariae*, n. gen., n. sp., horizontal-longitudinal section

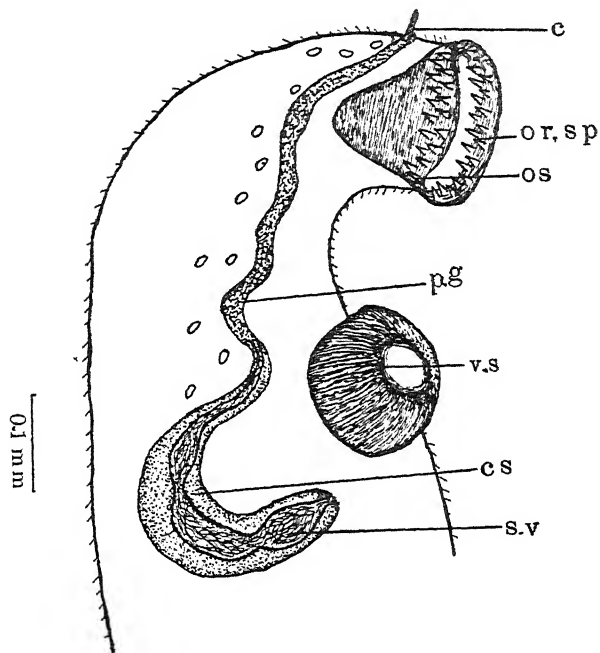


FIG 5.—*Masenia collata*, n. gen., n. sp., anterior part of the body showing cirrus sac.

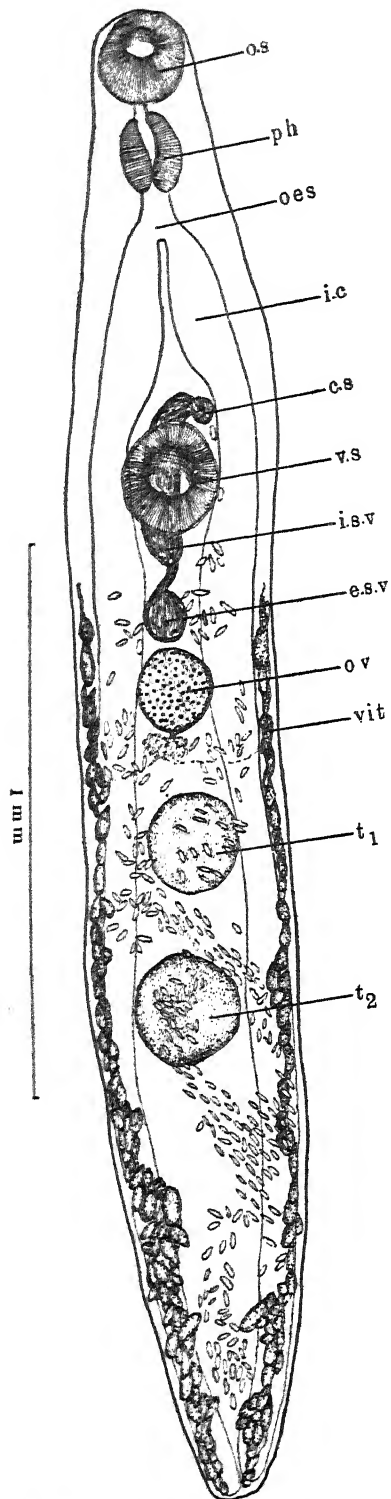


FIG 2.—*Ganada clariae*, n. gen., n. sp., ventral aspect.

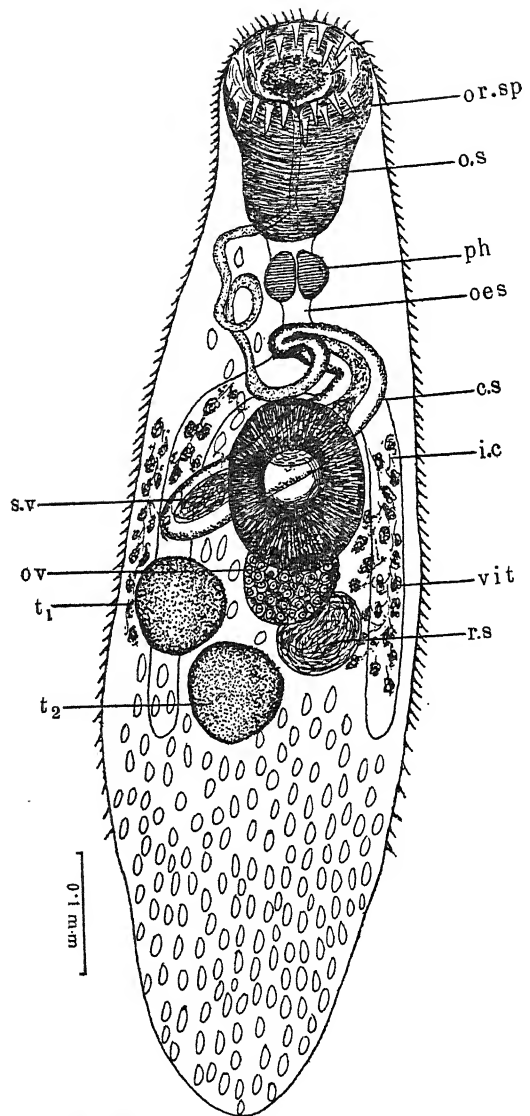


FIG 4.—*Masenia collata*, n. gen., n. sp., ventral aspect.

# ON NEW TREMATODES OF FROGS AND FISHES OF THE UNITED PROVINCES, INDIA.

**Part I.**—New Distomes of the family Hemiuridæ Luhe 1901 from North Indian fishes and frogs with a systematic discussion on the family Halipegidæ Poche 1925 and the genera Vitellotrema Guberlet 1928 and Genarchopsis Ozaki 1925.

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Communicated by Dr. H. R. Mehra

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## Introduction

In this paper are described a number of new distomes obtained from the gut of the frogs and fishes of Sitapur, Lucknow and Allahabad. Besides these forms a large number of other new species were obtained, which I shall describe in subsequent papers. The account of all these trematodes forms the subject of a thesis which I submitted in March 1933 in partial fulfilment of the requirements for the degree of Master of Science of the Allahabad University, and which for the purpose of publication I have split up into four parts. The first part as contained in this paper deals with two new species and a new variety of the genus *Halipegus* Looss 1899, two new species of the genus *Progonus* Looss 1899 and two new species of the new genus *Ophiocorchis*.

The systematic position of the family *Halipegidae* Poche 1925 and subfamily *Derogenetinae* Odhner 1927 of the family *Hemiuridae* Luhe 1901 is fully discussed and the author comes to the conclusion that the family *Halipegidae* is untenable and that the genus *Halipegus* should be included in the subfamily *Derogenetinae* of the family *Hemiuridae*. The synonymy of the genera *Vitellotrema* Guberlet 1928 and *Halipegus* Looss 1899 is clearly indicated. In the light of the new forms belonging to the genus *Progonus* which are described in this paper the genus *Genarchopsis* Ozaki 1925 is dropped and the species belonging to it is referred to the genus *Progonus*.

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Genus *Halipegus* Looss 1899

Vulpian in 1860 described *Distomum ovocaudatum*, parasitic in the mouth cavity and pharynx of European frogs. In 1872 Grebnitzky published an account of a new species of *Distomum* *Dist. kessleri* from the stomach of *Rana esculenta*. His paper was published in a rather obscure journal and therefore the parasite remained unknown till recently (1929). In 1890 Creutzburg worked the life history of *Dist. ovocaudatum*. This distome was also described independently by Sonsino and Looss in 1894. Five years later Looss created the genus *Halipegus* for *Dist. ovocaudatum*, assigning it somewhere near the *Syncoelinae*. In 1905 Stafford recognized *Dist. ovocaudatum* Nickerson 1898, parasitic in *Rana calasbiana* and *R. clamata* to be a new species which he called *Halipegus occidualis*. Ssinitzin who in 1905 worked out the life cycle of *Cercaria cystophora* described by Wagener in 1866 from *Planorbis marginata* found that the cyst passes into a dragon-fly larva—*Colopteryx virgo*, which serves as its intermediate host and the frogs feeding on these larvæ become infected with *H. ovocaudatum*. Poche in 1925 created a new family *Halipegidae* for the genus *Halipegus* and placed it under his superfamily *Hemiuroidea* Poche 1925. As will be seen from the systematic discussion the family *Halipegidae* is untenable. Guberlet in 1928 added a new genus *Vitellotrema* from the stomach of a water snake to the family *Halipegidae* Poche. This genus which is based only on the unlobed character of the vitellaria cannot be maintained and I accordingly assign it to the genus *Halipegus* Looss as *H. fusipora*. In the following year Isaitschikow described *H. rossicus* from the stomach of a Russian frog—*R. esculenta* and Bychowsky referred *Dist. kessleri* Grebnitzky to the genus *Halipegus*. Wlassenko in 1929 found specimens of *H. kessleri* in the stomach of *Natrix natrix* pointing out the close resemblance between them and *H. rossicus*. The latter form he regarded as synonymous with *H. kessleri*. Simer found in the intestine of *Polyodon spathula* *H. perplexus*, which he described in 1929. I add in this paper to the genus *Halipegus* two new species and a new variety from the stomach and intestine of Indian frogs.

***Halipegus mehransis* n. sp.**

(Fig. 1)

Host—*Rana cyanophlyctis*.

Habitat—Stomach.

Locality—Sitapur, Oudh (India).

This is the most common trematode infecting *R. cyanophlyctis* in Sitapur district from May to September. During this period the frogs were found to be invariably infected with this parasite. The average intensity of infection is about 4 although the number varies from 2—12 in a single host. The distomes live firmly attached to the wall of the stomach by means of their large

and powerful ventral sucker which, when in use, may be prominently protruded from the body. The free anterior end may be extended for some distance in a leech-like manner. The parasites have little power of adaptability to changed conditions of diet and temperature, for none could live for more than 28 hours in nutritive solutions kept at the laboratory temperature (*vide* Table I).

Table I

Date 2nd—6th August, 1932. Laboratory Temp. 70—82.5°F.

Nutritive solutions used.	Physiological salt solution 0.75%	Phys. salt sol. and yolk 1:2	Phys. salt sol. and albumen 1:1	Phys. salt sol. and yolk and albumen 1:2:2	5% sugar solution.
Number of parasites kept ...	5	5	5	5	5
Maximum number of hours they lived ...	6	20	18	28	8½
Number of parasites lived for maximum period ...	2	3	2	3	1

The parasites are light grey in colour and have enormous power of contraction and expansion. The thick and muscular body is spindle-shaped with bluntly pointed ends and a nearly circular cross-section. The bodywall is smooth and entirely devoid of spines. It may be thrown into circular folds which are more prominent with increased degree of contraction. These folds give the trematodes a ringed appearance in the living condition. The sexually mature worms when alive vary from 1.7—4.5 mm in length and 0.6—0.9 mm in breadth. In entire mounts the parasites measure 3.1—5.1 mm in length and 0.9—1.2 mm in maximum breadth which lies about the region of the acetabulum.

The suckers are well developed and highly muscular and have a circular outline. The subterminal and ventrally directed oral sucker of 0.28 mm diameter is nearly half the size of the acetabulum. The acetabulum of 0.5—0.72 mm diameter and 1.8 times the size of the oral sucker, situated in the middle of the body, is very deep extending nearly to the dorsal surface of the body.

The genital pore is ventral, usually median, rarely shifted slightly to one side; a little behind the intestinal bifurcation. The excretory pore lies terminally at the extreme hinder end of the body.

The pharynx, 0.1—0.14 mm in diameter, is situated just posterior and slightly dorsal to the oral sucker. The oesophagus being absent the intestinal caeca arise directly behind the pharynx. The caeca are broad and wavy with several marked constrictions and extend up to the extreme posterior end.

The massive testes are extra-caecal and have roughly triangular outline. They are situated obliquely, one on each side, close behind the anterior half of the body. The right testis,  $0.3-0.6 \times 0.27-0.43$  mm in size, lies in the space between the right intestinal caecum and the body wall, with its major portion in level with the posterior third part of the acetabulum. The left testis of  $0.3-0.6 \times 0.28-0.4$  mm size is situated  $0.09-0.17$  mm behind the acetabulum in the space between the left intestinal caecum and the body wall. The vesicula seminalis is a flask-shaped structure of  $0.22-0.25 \times 0.12-0.14$  mm size, situated slightly to the right of the median line, a little behind the intestinal bifurcation. The vesicula seminalis narrows anteriorly to form the ductus ejaculatorius,  $0.08 \times 0.01$  mm size, which bends downwards on the left side of the vesicula seminalis to open on a nipple-shaped cone or papilla lying in the genital atrium. The ductus ejaculatorius is surrounded by prostate gland cells which lie free in the parenchyma.

The ovary is nearly spherical in shape with entire margin, measuring  $0.16-0.27 \times 0.2-0.3$  mm in size. It is intra-caecal lying just in front of the left vitelline gland close to the median line. The oviduct arising from the middle of the posterior margin of the ovary turns towards the median line and after running for a short distance is joined first by the Laurer's canal and soon after by the common vitelline duct, before it enters the compact shell gland mass of an oblong form and  $0.17-0.25 \times 0.11-0.17$  mm size, situated obliquely to the right side in level with the ovary. The shell gland mass is separated from the latter by the anteriorly passing Laurer's canal. The Laurer's canal of  $0.03$  mm diameter has a number of transverse constrictions at regular intervals, throughout its length. A receptaculum seminis is absent. The relations of the female ducts are shown in figure 8.

The vitellaria lie in two groups, ventral to the intestinal caeca, one on either side of the median line close behind the ovary and the shell gland mass, ending posteriorly a little in front of the blind extremities of the intestinal caeca. The right vitelline gland,  $0.3-0.46 \times 0.2$  mm, in size, consists of four well marked lobes while the left one,  $0.27-0.35 \times 0.18-0.2$  mm in size, has five lobes. The vitelline duct of each side runs mesially and the two ducts meet in the median line to form the common vitelline duct just behind the shell gland mass.

The initial part of the uterus is filled with a huge number of sperms and hence may be regarded as receptaculum seminis uterinum. The uterus forms closely packed and irregularly arranged transverse convolutions which extend laterally up to the body wall both in front and behind the acetabulum. The uterine coils, however, never extend behind the shell gland mass and the vitellaria. In its terminal part the uterus lies parallel to the vesicula seminalis and like the ductus ejaculatorius is lined internally with cuticle. The uterus opens on the genital papilla very close to the male opening. It contains numerous eggs of golden yellow colour which bear a very long filament

at one end. The egg measures  $0.045 \times 0.018$  mm in size and filament 0.32 mm in length. The filament is 7 or 8 times the length of the egg.

The excretory bladder is Y-shaped, consisting of an unpaired excretory vesicle extending from the posterior end right up to the level of the testes where it divides into two lateral cornua. The cornua extend anteriorly, one on either side, uniting with each other on the dorsal side of the pharynx.

This species bears a very close resemblance to *H. occidualis* Stafford in the position of suckers, the extent of the intestinal caeca, lobed nature of the vitellaria, position of the gonads and the excretory pore. The important differences which mark it out as a new species are: the absence of the oesophagus, position of vitellaria and the genital pore, the union of the cornua of the excretory bladder in the region of the pharynx and not above the oral sucker, the size of the ova and the length of their filaments which are 7 or 8 times as long as the ovum and not shorter than the latter as in *H. occidualis*.

***Halipegus mehransis* var. *minutum* n. var.**

(Fig. 2)

Host—*Rana tigrina*.

Habitat—Stomach.

Locality—Sitapur, Oudh (India).

Eight specimens of this parasite were obtained from the stomach of two out of about sixty frogs examined during the rainy season in 1932. The distomes have a muscular and cylindrical body with marked power of contraction and expansion. In the living condition the parasites measure 0.9–1.9 mm in length and 0.36–0.7 mm in maximum breadth. Sexually mature worms in entire mounts measure 1.6–1.9 mm in length and 0.6–0.7 mm in maximum breadth across the testicular region.

The subterminal and ventrally directed oral sucker is transversely oval in outline, measuring  $0.14-0.2 \times 0.19-0.27$  mm in size. The acetabulum, situated about the end of the anterior half of the body, is spherical in outline with a diameter of 0.3–0.4 mm. It is one and a half times as large as the oral sucker. The pharynx is spherical with a diameter of 0.08–0.1 mm and opens directly into the two broad and wavy intestinal caeca which extend up to the extreme posterior end of the body.

The topography of the gonads and the structure of the copulatory apparatus are very much similar to those of *H. mehransis* n. sp. The testes, ovary, vitellaria and the shell gland complex all occupy the same relative positions as in the above species. The testes are extracaecal, lying somewhat asymmetrically one on each side, close behind on the sides of the acetabulum. The left testis,  $0.16-0.26 \times 0.11-0.17$  mm in size, lies a little cephalad of the right testis which measures  $0.12-0.25 \times 0.11-0.19$  mm in size. The vesicula seminalis of  $0.14-0.17 \times 0.08-0.096$  mm size is situated slightly to the right of the median line close behind the intestinal bifurcation. The

short ductus ejaculatorius of about 0.075 mm length is surrounded by a few prostate gland cells. The male pore lies on a small conical papilla situated in the genital atrium.

The ovary is a small spherical structure of 0.10–0.11 × 0.1–0.14 mm size, situated close to the right side of the median line just in front of the right vitelline gland. The shell gland complex, 0.096–0.14 × 0.06–0.08 mm in size, lies obliquely behind the ovary on the side opposite to that of the ovary just in front of the left vitelline gland. A conspicuous Laurer's canal is present but the receptaculum seminis is absent.

The vitellaria lie in two somewhat obliquely situated groups, one on each side close behind the ovary and the shell gland complex. The right vitelline gland of 0.14–0.22 × 0.096–0.19 mm size has five distinct lobes, while the left gland, 0.16–0.22 × 0.06–0.14 mm size, has only four such lobes. The vitelline ducts of both sides unite to form a common duct in the median line just behind the ootype.

The first crop of eggs produced are abortive. Two specimens of 1.6 mm length had the earliest eggs still in the uterine convolutions in the post-acetabular region. The most anterior eggs were about half the normal size and had irregular shape with very thin shell. The contents of the eggs were vacuolar in appearance. Fully mature specimens, however, have numerous golden yellow eggs in the transversely arranged coils of their uterus. The uterine convolutions are confined to the intercaecal area and never extend posterior to the ovary and the shell gland complex. Terminally the uterus opens on the genital papilla independently of the male opening. The eggs of 0.042–0.047 × 0.02 mm size have at one end a very long filament of 0.17–0.2 mm length. The excretory system is as in *H. mehransis*.

This trematode shows very close resemblance to *H. mehransis* in the general form, shape and the topography of the various organs, but differs from it in the smaller size of the body and of the various organs, transversely oval shape of the oral sucker, position and size ratio of the acetabulum and the size of the egg and its filament and the host. On the basis of these differences I consider this parasite to belong to a new variety of *H. mehransis* which I name *rar minutum* on account of its much smaller size.

### ***Halipegus spindale* n. sp.**

(Fig. 3)

Host – *Rana cyanophlyctis*.

Habitat – Intestine

Locality – Sitapur, Oudh (India).

Four mature specimens of this parasite were obtained from the intestine of a frog examined in July 1932 at Sitapur. The parasites have a smooth and spindle-shaped body nearly circular in cross-section. The specimens fixed

under a slight pressure measure 3.1—3.4 mm in length and 0.86—0.96 mm in greatest breadth which lies across the acetabular region. The genital atrium is situated a little behind the intestinal bifurcation slightly to the left of the median line. The excretory pore lies subterminally on the ventral surface near the hinder end of the body.

The suckers are well developed, muscular and have a circular outline. The oral sucker, of 0.26 mm diameter, is situated subterminally on the ventral surface. The acetabulum, 0.52 mm in diameter, is situated in the region between the anterior third and the first half of the body. The acetabulum is twice as large as the oral sucker.

The oral sucker leads posteriorly into a thick-walled spherical pharynx of 0.1—0.11 mm diameter which is followed by a very short oesophagus of 0.05—0.08 mm length. The intestinal caeca, with markedly crenated outline, are of uniform breadth, extending more or less in a straight course up to the anterior end of the vitellaria.

The testes are more or less ovoid in shape and lie asymmetrically outside the intestinal caeca, close behind the acetabulum. The left testis, 0.38—0.41 × 0.27—0.32 mm in size, is more cephalad, lying in part to the side of the acetabulum, while the right testis, 0.38—0.4 mm in size, is situated more caudad at about 0.17—0.2 mm distance behind the acetabulum. The vesicula seminalis of 0.21—0.22 × 0.1—0.12 mm size is a prominent bulb-shaped structure situated behind the intestinal bifurcation a little to the left of the median line. It is continued into a short bent ductus ejaculatorius of 0.13 mm length which opens on a short conical genital papilla situated in the shallow genital atrium. A number of prostate gland cells lie all round the ductus ejaculatorius.

The ovary, 0.16—0.17 × 0.19—0.22 mm in size, is intracaecal, situated to the right side in front of the right vitelline gland. The shell gland mass is an oval structure of 0.12 × 0.16 mm size, lying either in the median line or slightly to the left side close in front of the vitelline gland of the same side. The relations of the female genital ducts are as in *H. mehransis*.

The vitellaria lie in two groups, one on either side, at the extreme hinder end of the body and immediately behind the blind extremities of the intestinal caeca. The right vitelline gland of 0.3—0.32 × 0.24 mm size consists of five lobes while the left one measuring 0.3—0.32 × 0.16 mm in size has only four such lobes.

The uterus which is enormously developed and stuffed with numerous golden yellow eggs lies mostly in transverse convolutions extending up to the bodywall on either side. Terminally it opens on the small genital papilla situated in the shallow genital atrium. The eggs measure 0.045—0.047 × 0.02—0.022 mm in size and bear at their posterior end a long filament of 0.27 mm length.

The excretory system is as in *H. mehransis* with the difference that in this form the excretory pore lies subterminally on the ventral surface.

Of all the species of the genus *Halipegus spindale* bears a close relationship to *H. mehransis* in the form of the body, position of gonads and vitellaria, the relations of the female genital ducts and in the structure of the end apparatus of the reproductive organs. It differs, however, in the following important features which mark it out as a new species: the position and size ratio of the suckers, the presence of an oesophagus, the more or less straight and uniform breadth of the intestinal caeca ending in front of the vitellaria and the subterminal position of the excretory opening.

**Systematic discussion on the genus *Halipegus* Looss 1899 with remarks on the family *Halipegidae* Poche 1925, and the genus *Vitellotrema* Guberlet 1928.**

The systematic position of the genus *Halipegus* has been much debated upon by various workers. Looss, who created the genus in 1899, assigned it a place near the *Syncoeliinae*. Luhe in 1901 included it in the family *Hemiuridae*. Ward and Whipple in 1918 and Nicoll in 1926 placed it in the category of unclassified genera. Dollfus in 1923 and Viana in 1924 assigned it to the *Syncoeliinae*. In 1925 Poche, however, created for it a new family *Halipegidae* which he included in his superfamily *Hemiuroidea*. Guberlet in 1928 and Faust 1930 following Poche have maintained the family *Halipegidae*. Odhner in 1927 created a new subfamily *Derogenetinae* under the family *Hemiuridae*, for the genera *Halipegus*, *Derogenes*, *Gonocerca* and *Lecithophyllum* which he considered to be closely related. Fuhrmann in 1928 follows Odhner in assigning the genus *Halipegus* to the *Derogenetinae*.

The genera *Halipegus* and *Derogenes* are closely related on account of the marked similarity in the general body-form, position and size of suckers, length of the intestinal caeca, topography of the gonads and the vitellaria, position of genital pore, large size of eggs with a polar filament at the posterior end and in the excretory system. The only points of difference between the two genera are in the position and arrangement of uterine coils and the extent of the prostate glands—characters which can at the most be considered of generic importance. I, therefore, drop the family *Halipegidae* Poche 1925 and include the genus *Halipegus* in the *Derogenetinae*.

The genus *Vitellotrema* as included in the family *Halipegidae* by Guberlet differs from the type genus of the family only in the unlobed character of the vitelline glands. There is one species of *Halipegus*, i.e., *H. kessleri* syn. *H. rossicus* which has got unlobed vitelline glands like those of the genus *Vitellotrema*. It seems that Guberlet was not aware of the latter condition, as appears from the list of references given in his paper, otherwise he would not have thought of creating his new genus on the basis of this character.

The lobed or unlobed character of the vitelline glands, as discussed by Looss in 1901 and Manter in 1926, should not be considered to be of generic importance even in cases where the lobes are distinctly separated into closely aggregated follicles. This view is also supported by the condition of the vitelline glands in the new species of *Progonus* Looss and of *Ophiocorchis* n. gen. which are described by me in this paper. These species resemble each other closely in most features except in the lobed or unlobed character of the vitellaria. I, accordingly, drop the genus *Vitellotrema* and refer its type species to the genus *Halipegus*.

The diagnosis of the genus *Halipegus* as now constituted is as follows :--

*Derogenetinae*: with a highly muscular and smooth, usually cylindrical rarely flattened body. The suckers are well developed and muscular; the acetabulum larger than the oral sucker, situated about or in the middle of the body. Muscular pharynx present; oesophagus present or absent, intestinal caeca long extending either up to the extreme hinder end or stopping in front of the vitelline glands. The excretory bladder is Y-shaped with a long median stem and two long cornua which run forwards and unite together in the region of the oral sucker or the pharynx. The genital pore is situated either in the region of the pharynx or distinctly behind the intestinal bifurcation; a small genital atrium is present. A ductus hermaphroditicus may be absent or present. The testes two in number, situated symmetrically or asymmetrically in the first half of the post-acetabular region; a small vesicula seminalis and a slight pars prostatica are present but a cirrus is absent. The rounded ovary is situated near the hinder end of the body in front of the vitellaria. The vitellaria lie in two lobed or unlobed groups placed symmetrically or obliquely behind the ovary at the hinder end of the body. Receptaculum seminis is absent. Laurer's canal is present. The long uterus consists of only ascending part in transverse coils containing a huge number of large-sized eggs bearing a long or short polar filament at their posterior end. Parasitic in the mouth cavity, eustachian tubes, pharynx, stomach and intestine of fishes, frogs and snakes.

#### Key to the species of the genus *Halipegus* Looss.

- |  |   |                       |
|--|---|-----------------------|
| Vitelline glands lobed   | A |                       |
| Vitelline glands unlobed   | B |                       |
| A. Testes situated far behind the acetabulum, close to the ovary       |   | <i>H. ovocaudatum</i> |
| Testes situated close behind the acetabulum, far in front of the ovary |   | 1                     |
| 1. Oesophagus present  | 2 |                       |
| Oesophagus absent  | 3 |                       |



2. Intestinal caeca extend up to the extreme posterior end and the excretory pore terminal . . . *H. occidualis*  
Intestinal caeca end in front of the vitellaria, excretory pore subterminal . . . *H. spindale* n. sp.
  3. Genital pore lies in the region of the pharynx, the uterine coils do not overlap the intestinal caeca anteriorly in front of the testes . . . *H. longispina*  
Genital pore situated behind the intestinal bifurcation; uterine convolutions extend to the body-wall both in front and behind the acetabulum . . . 4
  4. Size 3.1–5.1 mm; acetabulum situated in the middle of the body 1.8 times the size of the oral sucker . . . *H. mehransis* n. sp.  
Size 1.9–1.9 mm; acetabulum situated between first 1/3 and 1/2 of the body and twice the size of the oral sucker . . . *H. mehransis* var *minutum* n. var.
- B. Intestinal caeca reach behind the vitellaria up to the extreme hinder end of the body . . . *H. fusipora*,  
Intestinal caeca stop in front of the vitellaria . . . *H. kessleri*.

### Genus *Progonus* Looss 1899 (= *Genarches*)

The only hitherto known species of this genus was described by Levinsen in 1881, for which Looss in 1899 created the genus *Progonus*, assigning it to the *Syncoeliinae* Looss. Luhe founded the family *Hemiuridae* in 1901 and included in it the genus *Derogenes* along with the genera with tail appendage. Looss in 1907 limited the scope of the family and retained under it only such forms as possess a tail appendage. Odhner in 1911 pointed out that *Derogenes* is so closely related to the other *Hemiuridae* that its separation from the family is impossible and that the genus *Progonus* which is closely related to *Derogenes* should be included in the *Hemiuridae*. Nicoll in 1913 agreed with Odhner in this view reducing the family *Hemiuridae* Looss to the position of a subfamily. Ozaki in 1925 described a new genus *Genarchopsis* a form closely resembling *Genarches mulleri* (Levins) and assigned it to the subfamily *Syncoeliinae*. Odhner in 1927 pointed out that *Progonus* shows a close relationship with *Derogenes* in most of its characters and consequently he included it with *Derogenes* in a new subfamily *Derogenetinae*. The only feature in which *Progonus* differs from *Derogenes* is the presence of a caudal anastomosis of the intestinal caeca near the hinder end of the body, which Odhner considers to be an example of "convergence". In the following year Fuhrmann following Odhner included the genera *Derogenes*, *Genarchopsis*, *Gonocerca*, *Licithophyllum*, *Bunocotyle*, and *Halipegus* in the subfamily *Derogenetinae*.

The systematic discussion at the end of the description of the new species of *Progonus* in this paper will show that the genus *Genarchopsis* Ozaki 1925 is identical with *Progonus* and that *P. ovocaudatum* is an intermediate species between the two synonymous genera.

***Progonus piscicola* n. sp.**

(Fig 4)

Host—*Ophiocephalus punctatus*.

Habitat—Stomach.

Locality—Allahabad.

Three specimens of this trematode were obtained from the stomach of one out of about a dozen fish examined in June 1932. In the living condition the parasites are light brown in colour and show great power of contraction and expansion. The body is muscular and somewhat cylindrical in form with a broadly rounded off anterior and a pointed posterior end. The distomes are of moderate size measuring 3.3–3.4 mm in size and 1.12 in maximum breadth which is attained about the middle of the body. The body in front of the acetabulum is uniformly broad while the post-acetabular portion tapers sharply to the posterior pointed end. The well-developed and muscular suckers have a circular outline. The oral sucker measuring 0.33–0.34 mm in diameter lies subterminally at the anterior end of the body, with its cavity directed towards the ventral surface. The acetabulum of 0.66–0.68 mm diameter is twice as large as the oral sucker, situated in the first half of the post-equatorial region.

The oral sucker opens posteriorly into a spherical thick-walled pharynx of 0.12–0.14 mm diameter. In the absence of an oesophagus the intestinal bifurcation takes place directly behind the pharynx at a distance of 0.48–0.53 mm from the anterior end. The intestinal caeca have a highly crenated outline and run at first transversely and then turning downwards continue in a wavy course up to the hinder end of the body where they are continuous into each other just in front of the vitellaria.

The excretory bladder is Y-shaped consisting of an unbranched median stem which bifurcates just behind the acetabulum into two long cornua extending laterally right up to the level of the pharynx and uniting with each other on the dorsal side of the latter. The excretory bladder opens terminally at the hinder end of the body. The terminal part of the bladder is surrounded by a sphincter formed by a group of deeply staining parenchymatous cells with prominent nuclei.

The semilunar slit-like genital pore is sinistral or median, situated ventrally in level with the pharynx. It leads into a roomy genital atrium of

left vitelline gland measures  $0.13-0.19 \times 0.05-0.08$  mm in size while the right one is of  $0.08-0.17 \times 0.09-0.1$  size.

This interesting species resembles *P. piscicola* n. sp. in the general body-form and size ratio of the suckers, absence of prepharynx and oesophagus, the end apparatus of the reproductive system and the lateral extent of the uterine coils. It differs, however, from the above species in the smaller size of body, position of the acetabulum, the course of the intestinal caeca, more caudal position of the testes, smaller size and position of the shell gland mass, the symmetrical position of the vitellaria and in the important fact that the uterine convolutions extend posteriorly beyond the shell gland mass and lie in the space between the two compact vitelline glands at the extreme hinder end of the body. In this last character this species resembles *P. mulleri*. (Levins.)

**Systematic discussion on the genus *Progonus* Looss (= *Genarches* Lss.)  
and on the genus *Genarchopsis* Ozaki 1925**

In discussing the diagnostic features of his new genus "*Genarchopsis*" Ozaki points out that his species bears a very close resemblance to *Genarches mulleri* (Levins) in the general shape of the body, the structure of the end apparatus of the reproductive organs and the excretory system but it differs in the convolutions of the uterus which do not stretch back further than the vitellaria. We find that *P. ovocaudatum* n. sp. resembles *Genarchopsis goppo* in nearly all features except in the extent of the uterus which extends further backwards than in the latter species, reaching up to the hinder end of the vitellaria or body. As the only distinction between the genera *Genarchopsis* and *Progonus*, i.e., in the extent of the uterus ceases to exist in my species *P. ovocaudatum*, the identity of *Genarchopsis* with *Progonus* becomes quite clear. I maintain that these two genera are identical and synonymous. This view is also supported by the condition of the uterine coils in the two species of *Ophiocorchis* n. gen. described in subsequent pages.

I assign the genus *Progonus* to the subfamily *Derogenetinae* Odhner 1927. I agree with Odhner that the posterior anastomosis of the intestinal caeca in *Progonus* must not be given undue systematic importance. The posterior intestinal anastomosis is present in many distantly related forms such as *Cyclocalium*, *Progonus*, *Opicoelous* and *Coitccoecum*, etc., and therefore must be considered as an example of convergence.

In view of what has been said above the diagnosis of the genus *Progonus* as given by Looss in 1899 needs a certain amount of modification. The amended diagnosis is as follows:—

Small distomes with elongated, flattened or cylindrical body tapering at both ends; suckers strongly developed; skin entire. Prepharynx present, oesophagus absent, intestinal caeca continuous into each other at the hinder

end of the body. Genital pore situated near the hinder end of the pharynx or behind the intestinal bifurcation; genital sinus is formed by the union of the male and female ducts and opens on a genital papilla; pars prostatica and the vesicula seminalis, however, not strikingly elongated and the latter does not reach the acetabulum towards its hinder end. Testes and ovary simple, rounded or oval. The vitellaria consist of two glands which may be lobed or compact, lying one on either side close behind the ovary. The uterine convolutions may or may not stretch back up to the hinder end of vitellaria. Excretory system as typical of the subfamily, *i.e.*, Y-shaped with the cornua uniting dorsal to the oral sucker or pharynx. Parasitic in the intestine and stomach of fresh-water and marine fishes.

### Key to the species of the Genus *Progonus* Looss 1899

- Uterus extends behind the shell gland mass reaching up to the posterior part of vitellaria ... 1
- Uterus does not extend posteriorly up to the vitellaria ... 2
- 1. Genital pore situated at the hinder end of pharynx ... *P. mulleri*.
- Genital pore situated a little behind the intestinal bifurcation ... *P. ovocaudatum*.
- 2. Uterine coils confined to intercaecal space ... *P. goppo*.
- Uterine coils not confined to the intercaecal space but extending to the bodywall on either side ... *P. piscicola*.

### *Ophiocorchis lobatum*. n. gen.; n. sp.

(Fig. 6)

Host—*Ophiocephalus striatus*.

Habitat—Stomach.

Locality—Lucknow.

In September 1932 I examined about thirty living specimens of *O. striatus* received from Lucknow, only two of which were found infected with two specimens each of this parasite. The distomes have a highly muscular and cylindrical body with smooth cuticle. In permanent mounts they measure 2·8—3·2 mm in length and 1·1—1·2 mm in breadth across the acetabular region. The well-developed suckers have a spherical outline. The oral sucker, 0·3 mm in diameter, occupies a subterminal position on the ventral surface. The acetabulum, measuring 0·76 mm in diameter, is 2·5 times the size of the oral sucker and is situated about the middle of the body.

Terminally the uterus passes into a well-developed muscular metraterm of  $0.15 \times 0.05$  mm size which after receiving the pars prostatica is continued as the ductus hermaphroditicus of 0.12 mm length. The ductus hermaphroditicus is capable of being protruded out of the 0.075 mm deep genital atrium and functions as a copulatory organ. The eggs measure  $0.035 \times 0.017$  mm in size and bear a polar filament of 0.012 mm length at one end.

*O. singularis* n. sp. bears a very close resemblance to the type species *O. lobatum* n. sp. The points of similarity are the presence of an oesophageal pouch, a metraterm, a pars prostatica and the topography of the gonads. It differs, however, from the latter species in the smaller size of the body, the extent of its uterine coils, position and size of the acetabulum and the compact nature of the vitelline glands.

I assign this new genus *Ophiocorchis* to the subfamily *Derogenetinae* of the family *Hemiuridae*. Of all the genera of the subfamily this interesting parasite in its affinities comes nearest to the genus *Progonus* Looss 1899 (as already amended by me in this paper). It shows some resemblance with the latter genus in the general body-form, position of the genitalia and the arrangement of the uterine coils. The remarkable points of difference which warrant the creation of a new genus for its reception are the presence of a well-developed globular pars prostatica, a large and highly muscular metraterm, a protrusible ductus hermaphroditicus capable of functioning as the copulatory organ and the presence of a peculiar structure which I have termed as the oesophageal pouch.

#### Key to the species of the genus *Ophiocorchis*

1. Vitelline glands lobed and the uterine coils do not extend in the region of the vitellaria ... *O. lobatum*.
2. Vitelline glands compact with the uterine coils extending between the two compact vitelline glands ... ... *O. singularis*.

#### EXPLANATION OF THE PLATES

- Fig. 1. Ventral view of *Halipegus mehransis*.  
 Fig. 2. Ventral view of *H. mehransis* var *minutum*.  
 Fig. 3. Ventral view of *H. spindale*.  
 Fig. 4. Ventral view of *Progonus piscicola*.  
 Fig. 5. Ventral view of *P. ovocaudatum*.  
 Fig. 6. Ventral view of *Ophiocorchis lobatum*.  
 Fig. 7. Ventral view of *O. singularis*.  
 Fig. 8. Diagrammatic view of female sexual organs of—*H. mehransis*.  
 Fig. 9. Diagrammatic view of female sexual organs of—*P. piscicola*.

## LETTERING

Act.	...	Acetabulum	Oot.	...	Ootype
C	...	Cirrus	Ph.	...	Pharynx
C. s.	...	Cirrus sac	P. ph	...	Prepharynx
Eg.	...	Egg	Pr. g.	...	Prostate glands
D. ej	...	Ductus ejaculatorius	P. p.	...	Pars prostatica
D. h.	..	Ductus hermaphroditicus	R. s.	...	Receptaculum seminis
E. b.	...	Excretory bladder	R. s u.	...	Receptaculum seminis uterinum
E. p.	...	Excretory pore	S. gl.	...	Shell gland
G. a.	...	Genital atrium	S. gl. c.	..	Shell gland complex
G. p.	...	Genital pore	T.	...	Testis
Int. c.	...	Intestinal caecum	Ut.	...	Uterus
L. c.	...	Laurer's canal	V. d.	...	Vas deferens
Mtm.	...	Metraterm	Vit.	...	Vitellaria
Oes.	..	Oesophagus	V. sm.	...	Vesicula seminalis
Oes. p.	...	Oesophageal pouch	V. s.	...	Ventral sucker
O. s	...	Oral sucker	Y. d.	...	Yolk duct
Ov.	...	Ovary	Y. r.	...	Yolk reservoir
O. d.	...	Oviduct			

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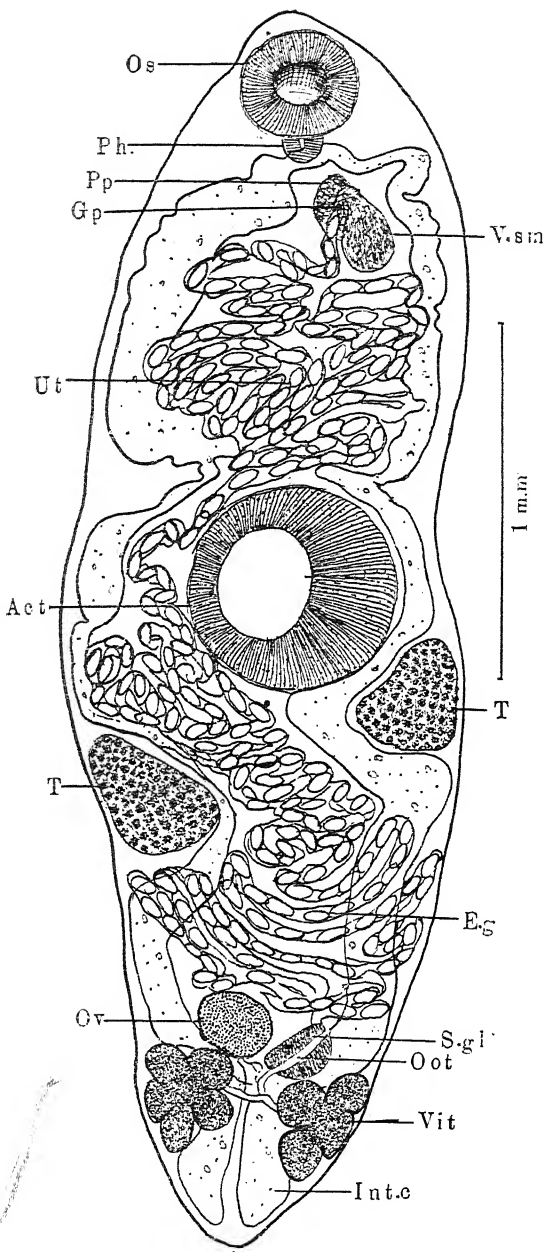


FIG. 1.—*Halipegus mehransis*, n. sp.

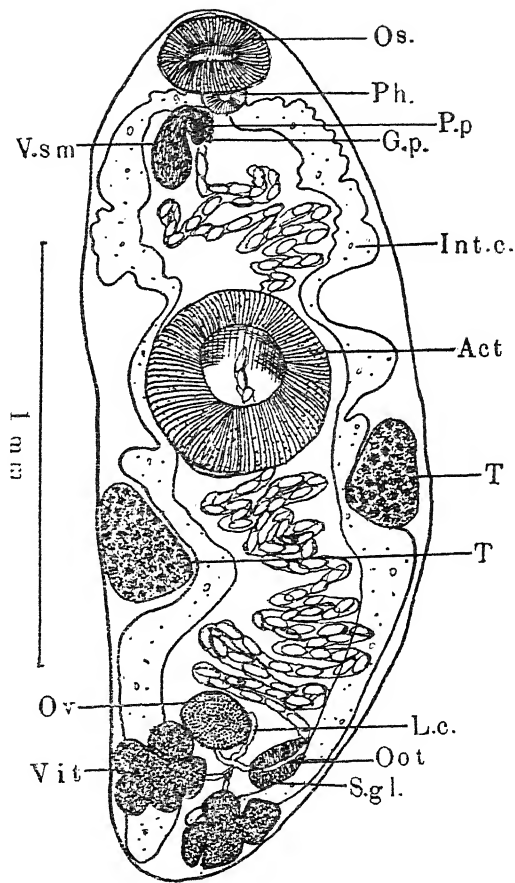


FIG. 2.—*Halipegus mehransis*, var *minutum*.



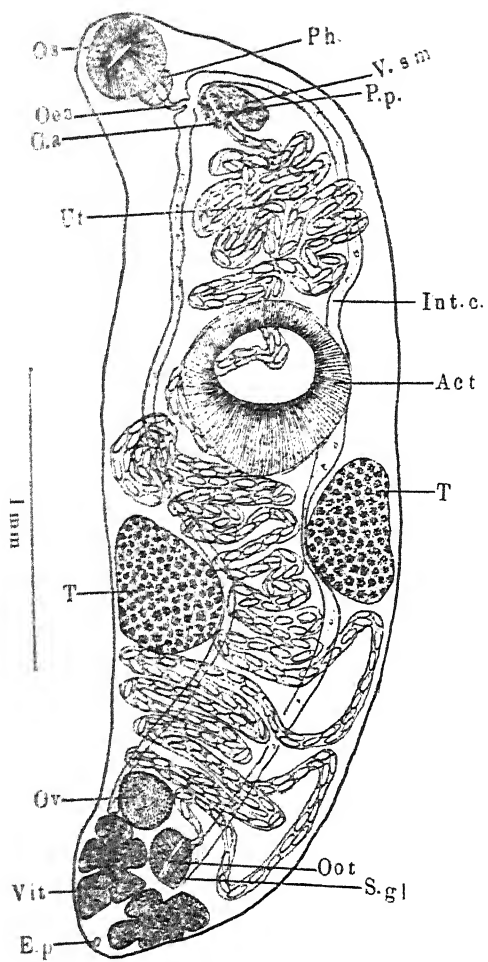


FIG. 3.—*Halipegus spindale*, n. sp.

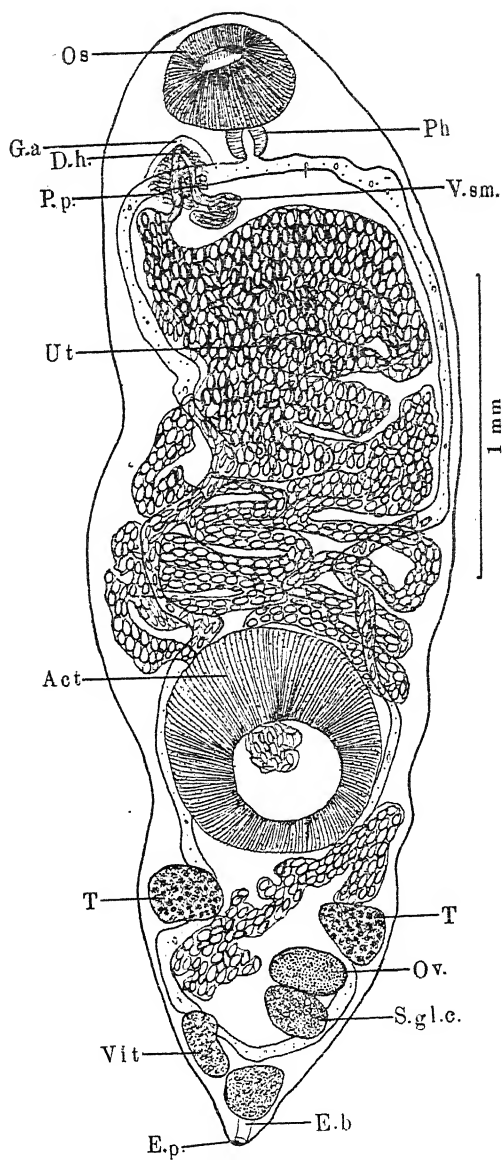


FIG. 4.—*Progonus piscicola*, n. sp.

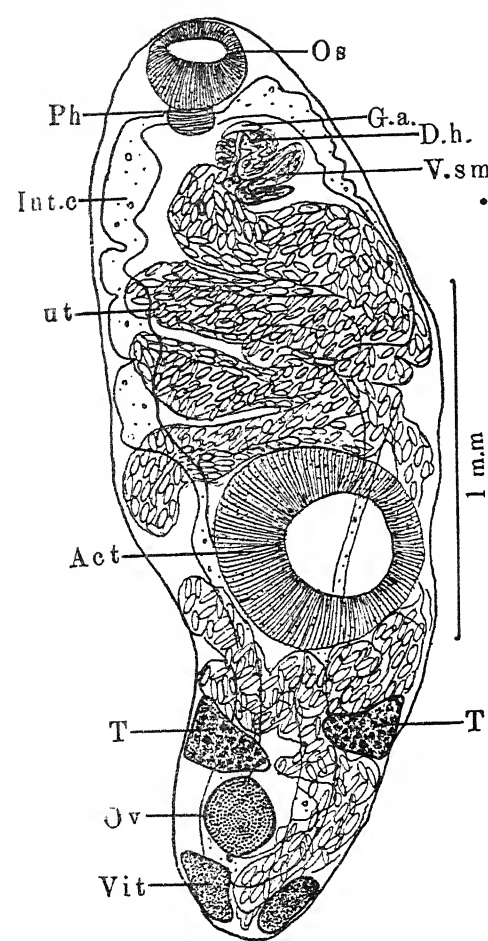


FIG. 5.—*Progonus ovocaudatum*

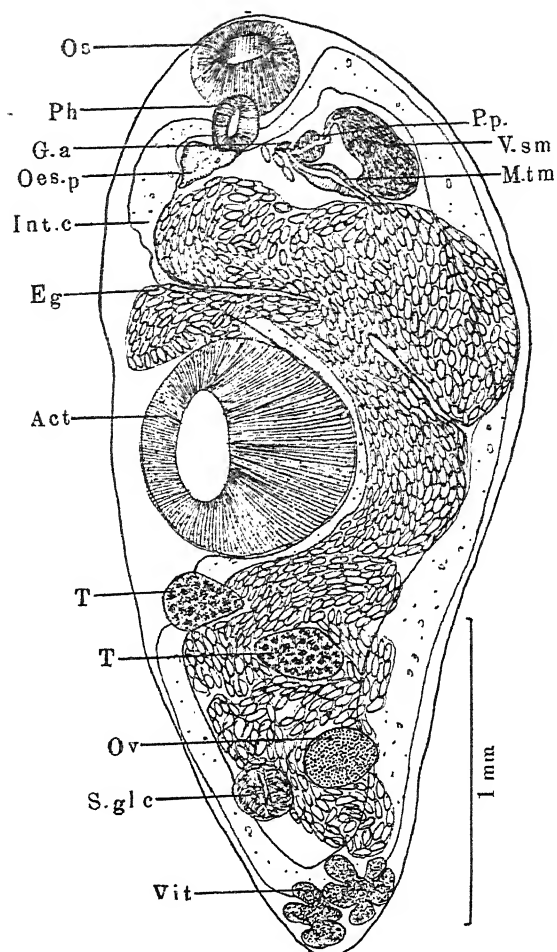


FIG. 6.—*Ophiocorchis lobatum*, n. gen., n. sp.

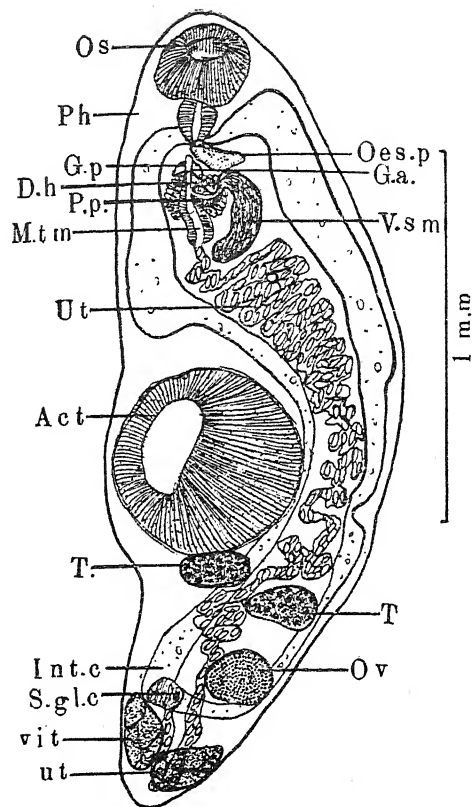


FIG. 7.—*Ophiocorchis singularis*

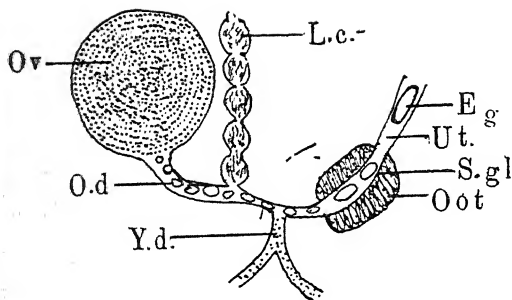


FIG. 8.

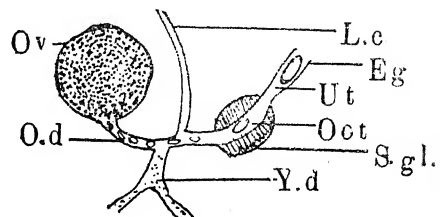


FIG. 9.

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DL. 3

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Number 1

### ON AN INFINITE SERIES OF INTEGRALS INVOLVING STURM—LIONVILLE EIGEN FUNCTIONS

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Communicated by Dr. P. L. Srivastava.

Received February 1, 1933.

#### INTRODUCTION.

In a previous paper<sup>1</sup> it has been proved by the writer that the series

$$\frac{\sum_n |a_{k,j}^{(n)}|}{k_1^2 k_2^2 \dots k_r^2 j_1^3 j_2^3 \dots j_s^3}, \text{ where}$$

$$a_{k,j}^{(n)} = j_1 j_2 \dots j_s \int_0^\pi \sin k_1 x \dots \sin k_r x \cos j_1 x \dots \cos j_s x \sin n x \, dx \text{ is}$$

uniformly convergent for all  $k_r, j_s$

Now  $\sin n x$  is just the Eigen function of a particular differential equation, viz.  $\frac{d^2 y}{dx^2} + \lambda y = 0$  for certain boundary values, viz.  $y(0)=0, y(\pi)=0$ , and it is natural to ask, therefore, whether a similar property is possessed by other Eigen-functions.

In another paper<sup>2</sup> the writer has shown that if  $\lambda_n (n=1, 2, \dots)$  are the Eigen-values and  $\phi_n(x)$  the corresponding Eigen-functions of the Sturm-Lionville Differential Equation

$$\frac{d}{dx} \left( p \frac{dy}{dx} \right) + \lambda y = 0$$

with the boundary conditions  $y(0)=0$ ,  $y(\pi)=0$ , and if we define

$$a_{j,k}^{(n)} = \int_0^\pi \phi_j(x) \phi_k(x) \phi_n(x) dx,$$

then the series  $\frac{\sum |a_{j,k}^{(n)}|}{\lambda_j \lambda_k}$  is uniformly convergent.

In the present paper, we prove the more general theorems that the series

$$\frac{\sum_{n=1}^{\infty} \sqrt{\lambda_n} |a_{j,k}^{(n)}|}{\lambda_j \lambda_k},$$

and

$$\frac{\sum_{n=1}^{\infty} \sqrt{\lambda_n} |b_{k,j}^{(n)}|}{\lambda_k \lambda_j^{\frac{3}{2}}},$$

also converge uniformly for all  $j, k \geq 1$ , where

$$b_{k,j}^{(n)} = \int_0^\pi \phi_k(x) \frac{d\phi_j}{dx} \phi_n(x) dx.$$

These Theorems are important in the theory of higher partial differential equations, as has been shown by the writer in a paper communicated to the London Math. Society.

#### §1.

Let  $p(x)$  be an essentially positive function defined in the interval  $0 \leq x \leq \pi$ , and let  $p(x)$  as well as  $\frac{dp}{dx}$  and  $\frac{d^2p}{dx^2}$  be continuous and uniformly bounded in the whole interval.

Let  $\lambda_n$  be the characteristic values and

$$\phi_n(x) \quad (n=1, 2, 3, \dots)$$

the characteristic functions of the Sturm-Liouville differential equation

$$(A) \quad \frac{d}{dx} \left( p \frac{dy}{dx} \right) + \lambda y = 0$$

with the boundary conditions

$$(B) \quad y(0)=0, \quad y(\pi)=0.$$

We assume that the characteristic functions are orthogonal and normalised.

For any pair of  $j, k$  we define a sequence of functions  $a_{j,k}^{(n)}$  by the relation

$$(1) \quad a_{j,k}^{(n)} = \int_0^\pi \phi_j(x) \phi_k(x) \phi_n(x) dx.$$

$$(n=1, 2, \dots)$$

and we shall prove the theorem that the series

$$(2) \quad \sum_{n=1}^{\infty} \frac{\sqrt{\lambda_n} |a_{j,k}^{(n)}|}{\lambda_j \lambda_k}$$

is uniformly convergent for all  $j, k$ .

The asymptotic expansions of  $\lambda_n$ ,  $\phi_n(x)$  and  $\frac{d\phi_n}{dx}$  for large  $n$  are known to be<sup>3</sup>)

$$(3) \quad \begin{aligned} \lambda_n &= n^2 \frac{\pi^2}{l^2} + O(1), \quad \sqrt{\lambda_n} = n \frac{\pi}{l} + O\left(\frac{1}{n}\right), \\ \phi_n(x) &= C_n \frac{\sin n p_2(x)}{\sqrt{p}} + O\left(\frac{1}{n}\right), \\ \frac{d\phi_n}{dx} &= C_n \frac{n\pi}{l} \frac{\cos n p_2(x)}{\sqrt{p}} + O(1), \end{aligned}$$

where

$$(4) \quad \begin{aligned} l &= \int_0^{\pi} \frac{1}{\sqrt{p}} dx, \\ p_2(x) &= \frac{\pi}{l} \int_0^x \frac{1}{\sqrt{p}} dx, \\ \frac{1}{C_n^2} &= \int_0^{\pi} \frac{\sin^2 n p_2(x)}{\sqrt{p}} dx. \end{aligned}$$

Now, since  $p(x)$  is essentially positive and  $\sin n p_2(x)$  does not vanish identically in  $0 \leq x \leq \pi$ , we deduce that  $\frac{1}{C_n}$  is greater than a + ve number  $c$  for all  $n$ , and therefore  $C_n$  is bounded. Consequently, the functions  $\phi_n(x)$  and  $\frac{1}{n} \frac{d\phi_n}{dx}$  are also bounded for all  $n$ .

Now, we have

$$a_{j,k}^{(n)} = \int_0^{\pi} \phi_j \phi_k \phi_n dx;$$

integrating by parts and considering that  $\phi_n$  satisfies the equation (A) and the condition (B), we get:

$$\begin{aligned} a_{j,k}^{(n)} &= -\frac{1}{\lambda_n} \int_0^\pi \phi_j(x) \phi_k(x) \frac{d}{dx} \left\{ p \frac{d\phi_n}{dx} \right\} dx, \\ &= -\frac{1}{\lambda_n} \int_0^\pi \phi_n(x) \frac{d}{dx} \left\{ p \frac{d}{dx} (\phi_j \phi_k) \right\} dx, \\ &= -\frac{1}{\lambda_n} \int_0^\pi \phi_n(x) \left[ \frac{d}{dx} \left( p \frac{d\phi_j}{dx} \right) \phi_k(x) + \frac{d}{dx} \left( p \frac{d\phi_k}{dx} \right) \phi_j \right. \\ &\quad \left. + 2 p \frac{d\phi_j}{dx} \frac{d\phi_k}{dx} \right] dx, \\ &= -\frac{1}{\lambda_n} \int_0^\pi \phi_n(x) \left\{ -(\lambda_j + \lambda_k) \phi_j(x) \phi_j(x) \phi_k(x) \right. \\ &\quad \left. + 2 p \frac{d\phi_j}{dx} \frac{d\phi_k}{dx} \right\} dx. \end{aligned}$$

Since  $\lambda_j, \lambda_k$  are  $> 1$  for all  $j, k$ , therefore we have

$$(5) \quad \frac{a_{j,k}^{(n)}}{\lambda_j \lambda_k} < \frac{2}{\lambda_n} \left\{ |a_{j,k}^{(n)}| + |d_{j,k}^{(n)}| \right\},$$

where

$$(6) \quad d_{j,k}^{(n)} = \int_0^\pi p(x) \frac{1}{\lambda_j} \frac{d\phi_j}{dx} \frac{1}{\lambda_k} \frac{d\phi_k}{dx} \phi_n dx$$

Therefore,

$$\begin{aligned} (7) \quad \left( \sum_n \sqrt{\lambda_n} \left| \frac{a_{j,k}^{(n)}}{\lambda_j \lambda_k} \right| \right)^2 &< \left( \sum_n \frac{2}{\sqrt{\lambda_n}} \left\{ |a_{j,k}^{(n)}| + |d_{j,k}^{(n)}| \right\} \right)^2 \\ &< \sum_n \frac{4}{\lambda_n} \sum_n \left\{ |a_{j,k}^{(n)}| + |d_{j,k}^{(n)}| \right\}^2 \end{aligned}$$

using the inequality of Schwarz. If we use further the inequality  $2ab \geq a^2 + b^2$ , then we get

$$(8) \quad \left( \sum_n \sqrt{\lambda_n} \left| \frac{a_{j,k}^{(n)}}{\lambda_j \lambda_k} \right| \right)^2 < \sum_n \frac{8}{\lambda_n} \sum_n \left\{ |a_{j,k}^{(n)}|^2 + |d_{j,k}^{(n)}|^2 \right\}$$

But we have

$$\begin{aligned} \phi_j(x) \phi_k(x) \phi_n(x) = & \left\{ C_j \frac{\sin j p_2}{\sqrt[4]{p}} + O\left(\frac{1}{j}\right) \right\} \left\{ C_x \frac{\sin x p_2}{\sqrt[4]{p}} + O\left(\frac{1}{k}\right) \right\} \\ & + \left\{ C_n \frac{\sin n p_2}{\sqrt[4]{p}} + O\left(\frac{1}{n}\right) \right\} \end{aligned}$$

and since all the  $C_n$  are bounded uniformly, we can write

$$(9) \quad \int_0^\pi \phi_j(x) \phi_k(x) \phi_n(x) dx < \alpha \int_0^\pi \frac{\sin j p_2 \sin k p_2 \sin n p_2}{p^{\frac{3}{4}}} dx + \beta \frac{1}{j k n},$$

where  $\alpha$  and  $\beta$  are constants properly chosen.

$$\text{Now } p_2(x) = \frac{\pi}{l} \int_0^x \frac{1}{\sqrt{p}} dx,$$

therefore

$$\frac{d p_2}{d x} = \frac{\pi}{l} \frac{1}{\sqrt{p}}, \quad p_2(0) = 0, \quad p_2(\pi) = \pi$$

So that

$$(10) \quad \int_0^\pi \phi_j(x) \phi_k(x) \phi_n(x) dx < \frac{\alpha l}{\pi} \int_0^\pi \frac{\sin j p_2 \sin k p_2 \sin n p_2}{\sqrt{p}} d p_2 + \beta \frac{1}{j k n}.$$

Therefore

$$\begin{aligned} (11) \quad \sum_n (a_{j,k}^{(n)})^2 & < \frac{\alpha^2 l^2}{\pi^2} \sum_n \left( \int_0^\pi \sin j p_2 \sin k p_2 \sin n p_2 \cdot \frac{1}{\sqrt{p}} d p_2 \right)^2 \\ & + \frac{2 \alpha l p}{\pi j k} \sum_n \frac{1}{n} \int_0^\pi \sin j p_2 \sin k p_2 \sin n p_2 \cdot \frac{1}{\sqrt{p}} d p_2 \\ & + \frac{\beta^2}{j^2 k^2} \cdot \sum_n \frac{1}{n^2}. \end{aligned}$$

The series

$$(12) \quad \sum_n \left( \int_0^\pi \sin j p_2 \sin k p_2 \sin n p_2 \cdot \frac{1}{\sqrt{p}} d p_2 \right)^2$$

is uniformly convergent for all  $j, k$ . Moreover,

$$\begin{aligned} & \sum_n \frac{1}{k j n^2} \int_0^\pi \sin j p_2 \sin k p_2 \sin n p_2 \cdot \frac{1}{\sqrt{p}} dp_2 \\ &= \sum_n \frac{1}{k j n^2} \int_0^\pi \cos n p_2 \frac{d}{dp_2} \left\{ \frac{1}{\sqrt{p}} \sin k p_2 \sin j p_2 \right\} dp_2 \\ &= \sum_n \frac{1}{n^2} \int_0^\pi \cos n p_2 \frac{d}{dp_2} \left\{ \frac{1}{\sqrt{p}} \frac{\sin k p_2}{k} \frac{\sin j p_2}{j} \right\} dp_2. \end{aligned}$$

For all values of  $j, k$ , the integral on the right is uniformly bounded, and therefore the series

$$(13) \quad \sum_n \frac{1}{k j n^2} \int_0^\pi \frac{1}{\sqrt{p}} \sin j p_2 \sin k p_2 \sin n p_2 dx$$

is uniformly convergent.

Also for all  $j, k$

$$\frac{1}{j^2 k^2} \sum_n \frac{1}{n^2} \leq \frac{\pi^2}{6}.$$

From (13), We see therefore that the series

$$(14) \quad \sum_n \left( a_{j, k}^{(n)} \right)^2$$

is uniformly convergent for all  $j, k$ .

$$\begin{aligned} \text{Now, } p(x) \phi_n(x) \frac{1}{\lambda_j} \frac{d\phi_j}{dx} \frac{1}{\lambda_k} \frac{d\phi_k}{dx} &= \left\{ C_n p^{\frac{3}{2}} \sin n p_2 + O\left(\frac{1}{n}\right) \right\} \\ &+ \left\{ \frac{C_j l}{\pi j} p^{-\frac{1}{2}} \cos j p_2 + O\left(\frac{1}{j^2}\right) \right\} \left\{ \frac{C_k l}{\pi k} p^{-\frac{1}{2}} \cos k p_2 + O\left(\frac{1}{k^2}\right) \right\} \end{aligned}$$

So that

$$(15) \quad |a_{j, k}^{(n)}| < \frac{\gamma}{k j} \int_0^\pi p^{\frac{7}{2}} \cos j p_2 \cos k p_2 \sin n p_2 + \delta \frac{1}{k^2 j^2 n^2}$$

where  $\gamma$  and  $\delta$  are two constants properly chosen.

Exactly as in the case of the series  $\sum_n \left( a_{j,k}^{(n)} \right)^2$ , it can be proved that also the series

$$(16) \quad \sum_n \left( d_{j,k}^{(n)} \right)^2$$

is uniformly convergent for all  $j, k$ .

Also, since  $\sum_n \frac{1}{\lambda_n}$  is evidently convergent, we see from (8) on account of (14) and (16) that the series

$$(17) \quad \sum_{n=1}^{\infty} \sqrt{\lambda_n} \frac{1}{\lambda_j \lambda_k} a_{j,k}^{(n)}$$

converges uniformly for all  $j, k$ .

§2.

In this paragraph, we shall prove that on defining the sequence of functions  $b_{k,j}^{(n)}$  more generally by the Integral.

$$(18) \quad b_{k,j}^{(n)} = \int_0^{\pi} \phi_k(x) \frac{d\phi_j}{dx} \phi_n(x) dx$$

a similar theorem holds, viz. that the series

$$(19) \quad \sum_{n=1}^{\infty} \frac{\sqrt{\lambda_n} 1}{\lambda_k \lambda_j^{\frac{3}{2}}} b_{k,j}^{(n)}$$

converges uniformly for all  $k, j$ .

Since  $\phi_n(x)$  satisfies the differential equation (A) we can write (18) in the form

$$(20) \quad b_{k,j}^{(n)} = -\frac{1}{\lambda_n} \int_0^{\pi} \phi_k \phi'_j \left( p \phi'_n \right)' dx,$$

where dashes are used to denote derivatives. Integrating by parts and remembering that  $\phi_n(x)$  satisfies the boundary conditions (B), we get

$$\begin{aligned} b_{k,j}^{(n)} &= \frac{1}{\lambda_n} \int_0^{\pi} \phi'_n p \left( \phi_k \phi'_j \right)' dx \\ &= \frac{1}{\lambda_n} \int_0^{\pi} \phi'_n \left\{ p \phi'_k \phi'_j - \lambda_j \phi_k \phi_j - p' \phi_k \phi_j \right\} dx. \end{aligned}$$

Integrating again by parts, we have,

$$\begin{aligned}
 b_{k,j}^{(n)} &= -\frac{1}{\lambda_n} \int_{\pi_0}^{\pi} \phi_n \left\{ \left( p \phi_k' \phi_j' \right)' - \lambda_j \left( \phi_n \phi \right)' - \left( p' \phi_k \phi_j \right)' \right\} dx \\
 &= -\frac{1}{\lambda_n} \int_{\pi_0}^{\pi} \phi_n \left\{ p' \phi_k' \phi_j' + p \phi_k'' \phi_j' + p \phi_k' \phi_j'' - \lambda_j' \phi_k' \phi_j \right. \\
 &\quad \left. - \lambda_j \phi_k' \phi_j' - p'' \phi_k \phi_j' - p' \phi_k' \phi_j' - p' \phi_k \phi_j'' \right\} dx, \\
 &= -\frac{1}{\lambda_n} \int_{\pi_0}^{\pi} \phi_n \left\{ \phi_j' \left( -\lambda_k \phi_k - p' \phi_k' \right) + \phi_k' \left( -\lambda_j \phi_j - p' \phi_j' \right) \right. \\
 &\quad \left. - \lambda_j \phi_k' \phi_j - \left( \lambda_j + p'' \right) \phi_k \phi_j' + \frac{p'}{p} \phi_k \left( \lambda_j \phi_j + p' \phi_j' \right) \right\} dx, \\
 (21) &= -\frac{1}{\lambda_n} \int_{\pi_0}^{\pi} \phi_n \left\{ \phi_k \phi_j' \lambda_j \frac{p'}{p} + \phi_k \phi_j' \left( -\lambda_k - \lambda_j - p'' + \frac{p'^2}{p} \right) \right. \\
 &\quad \left. - 2 \lambda_j \phi_k' \phi_j - 2 p' \phi_k' \phi_j' \right\} dx.
 \end{aligned}$$

Now we see that the integrand on the right contains a term  $\lambda_j \phi_n \phi_k \phi_j'$ , and since in the asymptotic expansion of  $\phi_j'$  a factor  $j \frac{\pi}{l}$  also comes, it will be obviously not sufficient to divide this term by  $\lambda_k \lambda_j$  if we want the quotient to be bounded for all  $j$ . On the other hand, a division by  $\lambda_k \lambda_j^{\frac{3}{2}}$  is quite sufficient for this purpose. Thus we have

$$(22) \quad \frac{|b_{k,j}^{(n)}|}{\lambda_k \lambda_j^{\frac{3}{2}}} < \frac{1}{\lambda_n} \left\{ |e_{k,j}^{(n)}| + |f_{k,j}^{(n)}| + 2 |g_{kj}^{(n)}| + 2 |p_{kj}^{(n)}| \right\}$$

where

$$e_{k,j}^{(n)} = \int_{\pi_0}^{\pi} \frac{p'}{p} \phi_k \phi_j \phi_n dx$$



$$\begin{aligned}
 (23) \quad f_{k,j}^{(n)} &= \int_0^\pi \left( 2 + p'' + \frac{p'^2}{p} \right) \phi_k \frac{\phi'_j}{\sqrt{\lambda_j}} \phi_n dx, \\
 g_{k,j}^{(n)} &= \int_0^\pi \frac{\phi'_k}{\lambda_k} \phi_j \phi_n dx, \\
 h_{k,j}^{(n)} &= \int_0^\pi p' \frac{\phi'_k}{\lambda_k} \frac{\phi'_j}{\lambda_j} \phi_n dx.
 \end{aligned}$$

Now, it is not difficult to prove exactly as for the series  $\sum_n \left( a_{j,k}^{(n)} \right)^2$  and  $\sum_n \left( d_{j,k}^{(n)} \right)^2$  in the first paragraph, that all the series

$$\begin{aligned}
 (24) \quad & \sum_n \left( e_{k,j}^{(n)} \right)^2, \quad \sum_n \left( f_{k,j}^{(n)} \right)^2 \\
 & \sum_n \left( g_{k,j}^{(n)} \right)^2, \quad \sum_n \left( h_{k,j}^{(n)} \right)^2
 \end{aligned}$$

are uniformly convergent for all  $k, j$ .

From (22) we get on account of Schwarz's inequality

$$\begin{aligned}
 (25) \quad & \left( \sum_n \frac{\sqrt{\lambda_n} |b_{k,j}^{(n)}|}{\lambda_k \lambda_j^{\frac{3}{2}}} \right)^2 < \sum_n \frac{1}{\lambda_n} \sum_n \left\{ |e_{k,j}^{(n)}|^2 + |f_{k,j}^{(n)}|^2 + 2 |g_{k,j}^{(n)}| + 2 |h_{k,j}^{(n)}| \right\}^2 \\
 & < \sum_n \frac{1}{\lambda_n} \sum_n \left\{ 6 |e_{k,j}^{(n)}|^2 + 6 |f_{k,j}^{(n)}|^2 + 12 |g_{k,j}^{(n)}|^2 + 12 |h_{k,j}^{(n)}|^2 \right\}
 \end{aligned}$$

using the inequality  $2ef \leq e^2 + f^2$ , etc.

From (24) and (25) we see therefore that the series

$$(26) \quad \sum_n \frac{\sqrt{\lambda_n} |b_{k,j}^{(n)}|}{\lambda_k \lambda_j^{\frac{3}{2}}}$$

converges uniformly for all  $k, j$ , which proves our theorem.

Finally, we remark that there should be no considerable difficulty in proving the following theorem along similar lines.

If

$$\begin{aligned}
 (27) \quad b_{k_1, k_2, \dots, k_r}^{(n)} &= \int_0^\pi \phi_n^{(x)} \phi_{k_1} \phi_{k_2} \dots \phi_{k_r} \phi'_{j_1} \phi'_{j_2} \dots \phi'_{j_s} dx \\
 & \quad j_1, j_2, \dots, j_s \\
 & \quad (n=1, 2, \dots)
 \end{aligned}$$

then the series

$$(28) \quad \sum_n \frac{\sqrt[n]{\lambda_n} \left| b_{j_1, \dots, j_s}^{(n)} \right|}{\lambda_{k_1} \lambda_{k_2} \dots \lambda_{k_r} \lambda_{j_1}^{\frac{3}{2}} \lambda_{j_2}^{\frac{3}{2}} \dots \lambda_{j_s}^{\frac{3}{2}}}$$

is uniformly convergent for all  $k_r$  and  $j_s$ .

#### References.

<sup>1</sup> M. R. Siddiqi, "Zur Theorie der nicht-linearen partiellen Differential-gleichungen vom parabolischen Typus," *Math. Zeitschrift*, 35, 475 (11), 1931.

<sup>2</sup> M. R. Siddiqi, On an infinite system of non-linear integral equations. *Bull. Cal. Math. Soc.*, 24, pp. 47-51, 1932.

<sup>3</sup> Courant—Hilbert: *Methoden der mathematische Physik I*, 2nd Ed (1931), pp. 290-291.

<sup>4</sup> M. R. Siddiqi, "Zur Theorie der nicht-linearen partiellen Differential-gleichungen vom parabolischen Typus," *Math. Zeitschrift*, 35, 476 (16), 1931.

# A CLASS OF DIRICHLET'S SERIES POSSESSING ESSENTIAL CHARACTERISTICS OF A TAYLOR'S SERIES

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1. In case of a Taylor's series its circle of convergence is also its circle of absolute convergence, and contains at least one singular point of the function represented by the series. As is well-known, no such simple relation holds in the case of general Dirichlet's series. Indeed a Dirichlet's series convergent in a portion of the plane may be absolutely convergent in a smaller region, and may represent a function all over the plane or in a wider region of it. For example, in the case of the series  $\sum_{n=1}^{\infty} (-1)^{n-1} n^{-s}$ ,  $\sigma_0 = 0$ ,  $\sigma = 1$ , and the function represented by the series is an integral function of  $s$ . There is, however, one important class of Dirichlet's series, namely, the series all of whose coefficients are positive, for which the line of convergence is also the line of absolute convergence, and contains at least one singularity of the function represented by the series.<sup>1</sup> My main object in this paper is to point out the existence of another class of Dirichlet's series for which the lines of convergence and absolute convergence coincide and necessarily contain at least one singularity of the function represented by the series. The result obtained in this direction is embodied in the following theorem.

## 2. THEOREM I.—If

(2.1)  $\lambda(z)$  be a branch of an analytic function of  $z (=x+iy=\rho+ \rho e^{i\phi})$ ,  $p-1 < \beta < p$  in the angle  $|\phi| \leq \alpha$ ,  $\alpha > 0$ , and  $\lambda(z) = o(\rho)$  uniformly in this angle as  $\rho \rightarrow \infty$ ;

(2.2)  $\lambda(x)$  be an  $L$ -function<sup>2</sup> such that it is positive for  $x \geq p$  and steadily tends to infinity with  $x$ ;

(2.3)  $\lambda'(z) = O(1)$  as  $\rho \rightarrow \infty$  uniformly in the angle  $|\phi| \leq \alpha$ ;

(2.4)  $f(\xi)$  be an analytic function of  $\xi = re^{i\theta}$  in the angle  $|\theta| \leq \alpha$ ,  $\alpha_1 > 0$ , and satisfy the relation  $f(\xi) = O(e^{Mr})$ , throughout this angle;

Also  $|F(s + \rho e^{ia})| < K_e e^{(M + |s|)|\lambda(z)|} < K_e e^{(M + |s|)\epsilon\rho}$ , by virtue of hypotheses (2.1), (2.3), (2.4), where  $\epsilon$  is an arbitrarily small positive number and  $\rho$  is sufficiently large.

Hence the integral (2.11) converges like

$$\int_0^\infty e^{-\rho(2\pi \sin \alpha - (M + |s|)\epsilon)} d\rho$$

which is uniformly convergent for all bounded values of  $s, \epsilon$  being arbitrarily small.

Similarly we can prove that  $I_3$  represents an integral function of  $s$ .

As regards the integral  $I_1$ , it is equal to

$$\int_0^\infty \lambda'(x) f\{\lambda(x)\} e^{-s\lambda(x)} dx = \int_0^\infty f(\xi) e^{-s\xi} d\xi + \text{an integral function of } s.$$

So that

$$(2.12) \quad H(s) - J(s) = G(s),$$

where  $G(s)$  is an integral function of  $s$ , and  $J(s)$  is an analytic function of  $s$

defined initially by the integral  $\int_0^\infty f(z) e^{-sz} dz$ .

The equation (2.12) has been obtained on the assumption that  $s$  is real, positive and sufficiently large. But as the right-hand side represents an integral function of  $s$  the equation persists for all values of  $s$ . That is, *the finite singularities of  $H(s)$  are identical with those of  $J(s)$* .

Now by virtue of a result<sup>3</sup> established by me and Mr. Jain recently,  $J(s)$  is an analytic function of  $s$  in the region lying exterior to a convex curve  $\Sigma$  which is the envelope of the lines (2.8) each of which contains at least one singularity of  $J(s)$ . The same is, therefore, true of  $H(s)$ , and in particular

$$\sigma = \sigma_0 = \lambda(0) = \lim_{r \rightarrow \infty} \frac{\log |f(r)|}{r}, \text{ and the line of convergence contains at least}$$

one singularity of  $H(s)$ .

If  $\lambda(\theta)$  is  $-\infty$  for any value of  $\theta$ ,  $J(s)$  is an integral function of  $s$ , and so is  $H(s)$ .

This completes the proof of our theorem.

#### REMARKS

3. Theorem I enables us to study the singularities of a class of Dirichlet's series in terms of those of the Laplace-Abel integral. Now we propose to make a few observations on this theorem.

(a) The function  $\lambda(z)$  contemplated in the theorem may be any one of a class of functions such as  $\frac{z}{\log z}$ ,  $\frac{z}{\log \log z}$ ,  $z^a$  ( $0 < a < 1$ ),  $\sqrt{z} \log z$ ,  $(\log z)^\beta$ ,  $(\log \log z)^k$ , etc. It will be noticed that the hypotheses (2.1), (2.2), (2.3) and (2.5) are all true for such a  $\lambda(z)$ . If  $\lambda(z) = z$ , we have the case of a Taylor's series in  $e^{-s}$ . If  $\frac{\lambda(n)}{n} \rightarrow \infty$  as  $n \rightarrow \infty$ , the line of convergence of a Dirichlet's series is, in general, a singular line.

(b) In the coefficient of the  $n$ th term  $\lambda'(n)$  is an essential factor for the truth of the theorem. Suppose  $\lambda(z) = \log \log z$ , and  $f(z) = 1$ . Then the theorem fails for the series  $\sum f\{\lambda(n)\} e^{-s\lambda(n)}$ , but holds for  $\sum \lambda'(n) f\{\lambda(n)\} e^{-s\lambda(n)}$ .

(c) If  $f(z)$  is an integral function of  $s$  of exponential type, then  $H(s)$  is an analytic function of  $s$  outside a closed convex curve  $\Sigma$ . In particular if  $f(z) = \sum_0^\infty \frac{c_\nu z^\nu}{\nu!}$  and the series  $\sum_0^\infty c_\nu z^\nu$  has the circle of its convergence  $|z| = \frac{1}{k}$  as a singular line, then the circle  $|s| = k$  is a singular line for the

function  $H(s)$ , since  $H(s) = G(s) + \sum_0^\infty \frac{c_\nu}{s^{\nu+1}}$ .

Suppose  $H(s) = \sum \frac{\sin \{\pi \log \log n\}}{n \log n} e^{-s \log \log n}$ . Then  $H(s)$

has singular points at  $s = \pm \pi i$ , and with the possible exception of the points lying on the line joining these two singular points  $H(s)$  is analytic everywhere else in the plane.

(d) If  $f(z)$  is analytic in the half-plane  $R(z) \geq 0$ , and

$$\lambda(\theta) = \theta \sin \theta + \cos \theta \log \cos \theta, \quad \left( |\theta| \leq \frac{\pi}{2} \right),$$

then the curve  $\Sigma$  is given by  $\sigma = \log \cos t$ ,  $\left( |t| \leq \frac{\pi}{2} \right)$ ,

and is a singular line for the function  $H(s)$ .

4. Appealing to another result of the paper <sup>6</sup> already referred to, we can deduce the following theorem:—

**THEOREM II.** *If in theorem I,  $\alpha_1 > \frac{\pi}{2}$ , and  $f(z)$  is such that  $f[\lambda(n)] = 0$  for  $n = p, p+1, p+2, \dots$  then  $f(z) \equiv 0$ .*

For  $H(s)$  is now identically zero, and so  $J(s)$  must be an integral function of  $s$ . But, since  $a_1 > \frac{\pi}{2}$ ,  $J(s)$  is identically zero, and so  $f(z) \equiv 0$ .

### References

- <sup>1</sup> Hardy: *The General Theory of Dirichlet's Series*, Theorem 10.
- <sup>2</sup> Hardy: *Orders of Infinity*, p. 17.
- <sup>3</sup> Hardy: *Orders of Infinity*, p. 34.
- <sup>4</sup> Whittaker and Watson: *Modern Analysis*, Third Edition, p. 71.
- <sup>5</sup> Srivastava and Jain: *Bul. Acad., Sci. U. P.* 2, 60, 1932.
- <sup>6</sup> The same reference as 5, p. 63.

# ON THE ABSORPTION SPECTRA OF PbO AND PbS

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The object of the present experiments is to extend Frank's work on the absorption spectra of saturated alkali halides to a new group of saturated compounds, *viz.*, the oxides and sulphides of di-valent and poly-valent atoms. Very little work has been reported on the absorption spectra of compounds of this type. Experiments on  $\text{SO}_3$  by A. K. Dutta,<sup>1</sup> on  $\text{N}_2\text{O}_5$ ,  $\text{MoO}_3$ , and  $\text{TeO}_3$ , by A. K. Dutta, and P. K. Sen Gupta,<sup>2</sup> and  $\text{CdO}$  and  $\text{ZnO}$  by P. K. Sen Gupta<sup>3</sup> have already been reported. In the present paper experiments on the absorption spectra of  $\text{PbO}$  and  $\text{PbS}$  are reported.

## EXPERIMENTAL

Owing to the high melting points of these substances, *viz.*, about  $875^\circ\text{C}$ . for  $\text{PbO}$  and  $1100^\circ\text{C}$  for  $\text{PbS}$  the vacuum graphite furnace of this laboratory was used for heating these substances. At first the salts were put on a silica tube; but on heating it was found that lead glass was easily formed. So the salts were put on asbestos pieces rolled into tubular form which was then inserted in the graphite tube. Asbestos was found not to give any appreciable vapour pressure at the temperatures required. In order to prevent the dissociation of  $\text{PbO}$  the furnace was evacuated and oxygen at a pressure of 50 cm. was filled in. In the case of  $\text{PbS}$  the spectra were taken first with the furnace completely evacuated and then with nitrogen at a pressure of 50 cm. This was necessary to prevent the oxidation of sulphur evolved by the dissociation of  $\text{PbS}$ .

The source of continuous light was a hydrogen tube run by a 2 KW Transformer. Photographs were taken by an  $E_3$  quartz spectrograph; the time of exposure being from 2—5 minutes. Copper arc was used for comparison.

## RESULTS

### PbO [yellow]

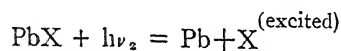
At lower temperatures the 2833A line of lead was prominent indicating that there was a partial decomposition of the oxide. As the temperature was further increased this line broadened out and at about  $950^\circ\text{C}$  merged into continuous absorption. This begins with a few bands at 3000 A.U. There is a retransmission followed by a second cut at 2240 A.U.

### PbS

The absorption spectra shows a continuous absorption also beginning with a few bands at 3500 A.U. A retransmission in this case also follows with a second cut at 2450 A.U.

## CALCULATION

The photo-chemical reaction may be represented by the following equation; since continuous absorption according to Frak-Condon principle indicates the breaking up of the molecule into constituent atoms,  $\text{PbX} + h\nu_1 = \text{Pb} + \text{X}$ : where X stands for O or S atom. The dissociation may, however, also occur as



where  $h\nu_2 - h\nu_1$  is the excitation energy of X.

From Born cycle we get

$$R = h\nu = Q + \frac{1}{2} D_{X_2} + L_{\text{Pb}} - L_{\text{PbX}} + L_X \dots (1)$$

Where  $Q$  = the latent heat of formation of  $[\text{Pb X}]$  from  $[\text{Pb}] + \frac{1}{2} [\text{X}_2]$

$D_{X_2}$  = the heat of dissociation of  $\text{X}_2$

$L_{\text{Pb}}$  = the latent heat of Pb.

$L_{\text{PbX}}$  = " " " "  $\text{PbX}$ .

$L_X$  = " " " " X.

The thermo-chemical data involved in this equation were taken from the tables of Landolt and Börnstein. They are tabulated below together with the various values for R.



TABLE 1. PbS

Limit of Absorption.	R K cal	Q K cal	DS.* K cal	L <sub>Pb</sub> K cal	L <sub>S</sub> K cal	L <sub>PbS</sub> K cal
- 3500A	80.2	22	56.7	44.5	15	55.6
2450A	116.7					

The value 55.6 K cal. for L<sub>PbS</sub> was obtained with the help of the relation  $L = R \frac{T_1 T_2}{T_2 - T_1} \log_e \frac{P_2}{P_1}$  from the following vapour pressure data given by Schneek and Albers<sup>2</sup>

t	p	t	p
850°C	2 mm	968°C	10.5 mm
917°C	4 mm	995°C	17 mm

The first reading for vapour pressure gives widely different values for the latent heat when combined with the others hence it was discarded. The other three gave consistent values; the mean being 55.6 K cal.

TABLE 2. PbO

Limit of Absorption	R K cal	Q K cal	$\frac{1}{2}$ DO. K cal	L <sub>Pb</sub> K cal	L <sub>PbO</sub> K cal
3000A	95.8	52.7	64	44.5	?
2240A	127.7				

No vapour pressure data for PbO are available. Hence its latent heat cannot be calculated.

PbS Applying the equation (1) to PbS we get

$$80.2 = 22 + 56.7 + 15 + 44.5 - L_{PbS}$$

or  $L_{PbS} = 58.0$  K cal.

$$*S_2 + 90 = 2S$$

$$4S_6 - 29 = 3S_3$$

$$S_6 + 64 = 3S_2$$

$$S_3 + 455 = 8S.$$

These data are taken from the tables of Landolt and Bornstein.

This agrees well with the value obtained from vapour pressure data, *viz.*, 55.6 K cal. The second cut is 1.59 volts higher than the first and may correspond to the excitation energy of sulphur from the state  $3P_0$  to  $1D_2$ . This has been found by Christie and Norde<sup>8</sup> to be 1.6 volts.

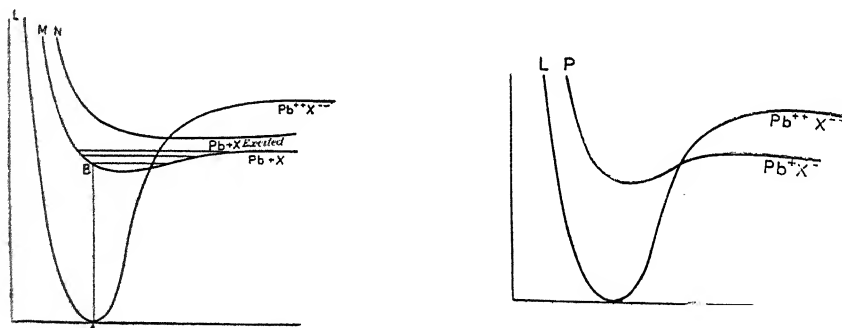
**PbO** Using the relation (1) we get  $95.3 = 52.7 + 64 + 44.5 - L_{\text{PbO}} - L_{\text{PbO}}$  comes out to be 66 K cal. This is a probable value since the latent heats of oxides are generally higher than those of the corresponding sulphides.

The difference between the first and the second cut is 32.4 K cal; which may correspond to the energy required to excite the oxygen atom from  $3P_0$  state to  $1D_2$  state; which is 1.9 volts.

### CONCLUSION

In the normal state the molecule may be assumed to be  $\text{Pb}^{++} \text{X}^{--}$  extending Frank's view as suggested by P. K. Sen Gupta. O has the same configuration as Neon and S as Argon. The ground state of  $\text{Pb}^{++}$ , which has the same configuration of electrons as Mercury is  $1S_0$ . The salts should be both diamagnetic.

Corresponding to the fact that the molecules are quite stable the potential energy curve will have a deep minima. It will be of the type L represented in the figure 1.



L to P will give rise to band absorption. Attempts to search for these probable bands are being made.

On the assumption that the normal state is  $\text{Pb}^{++} \text{X}^{--}$  the binding must obviously be electrostatic. A study of the lower half of Born cycle seems to lend some support to this view.

Here  $E$  = the lattice energy of the crystals.

$I_1$  = the ionisation potential of Pb

$I_2$  = the ionisation potential of  $\text{Pb}^+$ .

$I_e$  = two electron affinity of X

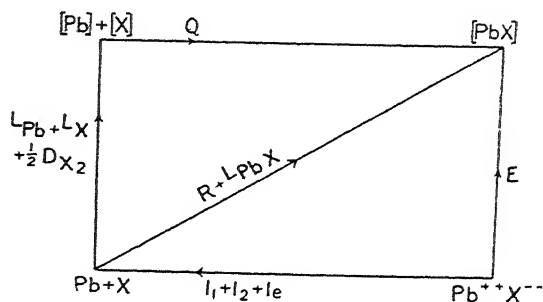


Fig. 3

All other symbols have already been defined.

From the lower half of Born cycle we get

$$E - (I_1 + I_2 + I_e) = R + L_{\text{PbX}} \quad (2)$$

#### CALCULATION OF $E$

The value of  $E$  for PbS has been calculated by Born and Gerlach<sup>11</sup> with the help of the formula

$$E = K \cdot \frac{n-1}{n} \sqrt[3]{\rho_{\text{IM}}}$$

where  $K$  = a constant involving Madelungs constant

$n$  = repulsion exponent

$\rho$  = density

$M$  = molecular weight.

The repulsion exponent  $n$  is given as  $n = 1 + 8.00 \times 10^{-11} \cdot \frac{1}{\chi} \left( \frac{M}{P} \right)^{\frac{4}{3}}$ ;  $\chi$  being the compressibility.

The lattice energy of PbS whose crystals are cubical of Sodium chloride type has been found by these authors as 636K cal.

The lattice energy of PbO unfortunately cannot be calculated; since there is no data for its compressibility.

The  $^3\text{P}_0$  state of Pb = 59810<sup>7</sup>

$^2\text{P}_{\frac{1}{2}}$  state of  $\text{Pb}^+$  = 121256<sup>6</sup>

Hence  $I_1 + I_2 = 517\text{K cal.}$

The double electron affinity of S has been calculated by Samuel and Lorentz<sup>1</sup> and they find it as  $-30$  K cal.

The left hand side of equation (2) thus gives

$$636 - (517 - 30) = 149 \text{ K cal}$$

while the right hand side gives  $R + L_{\text{PbS}} = 137.1 \text{ K cal.}$

The agreement is fairly good considering the uncertainties involved in the calculation of the lattice energy and the electron affinity of PS.

My thanks are due to Prof. M. N. Saha, F.R.S., for his kind interest in this work.

#### SUMMARY

1. The absorption spectra of PbO and PbS has been studied. There is a continuous absorption in both the cases showing that the photo-chemical reaction results in the dissociation of the molecules into two free atoms.

2. Retransmission in each case is interpreted as due to the excitation of sulphur or oxygen atom from the normal state  $^3P_0$  to the metastable state  $^1D_2$ .

3. The latent heat of PbS is found to be  $58.0$  K cal; agreeing well with the value obtained from vapour pressure data, *viz.*,  $55.6$  K cal. The latent heat of PbO is found to be  $66$  K cal.

4. A study of the Born cycle lends some support to the view that in the normal state the molecules are  $\text{Pb}^{++}\text{X}^{--}$ .

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- <sup>2</sup> Dutta, A. K. and Sen Gupta, P. K., *Proc. Roy. Soc.*, 139, 397 (1933).
- <sup>3</sup> Sen Gupta, P. K., *Bul. Acad. Sci. U. F.*, 2, 245 (1933).
- <sup>4</sup> Schenk and Albers, *Landolt and Börnstein Tables*.
- <sup>5</sup> Christie and Norde, *Phys. Rev.* 37, 903 (1930).
- <sup>6</sup> Born and Gerlach, *Zs. F. Phys.* 5, 441 (1921).
- <sup>7</sup> Grotrian and Geisler, *Zs. F. Phys.* 39, 377 (1926).
- <sup>8</sup> Geisler, *Zs. F. Phys.* 42, 265 (1927).
- <sup>9</sup> Samuel and Lorentz, *Zs. F. Phys.* 59, 59 (1930).

# THE ABSORPTION SPECTRA OF THE VAPOURS OF THE LOWER CHLORIDES OF ELEMENTS OF THE FIFTH GROUP OF PERIODIC TABLE

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Communicated by Prof. M. N. Saha.

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The object of the present paper is to report experiments on the absorption spectra of the trivalent halides of elements of the fifth group, *viz.*, P, As, Sb and Bi. As is well known they form pentahalides as well as trihalides. All available information about them is collected in the following table.

TABLE I.

Compound		Trichloride	Tribromide	Triiodide	Pentachloride	Pentabromide
Element						
Phosphorus	...	$\text{PCl}_3$ Colourless liquid b.p. $76^\circ\text{C}$ m.p. $-112^\circ\text{C}$ .	$\text{PBr}_3$ Colourless liquid b.p. $170.8^\circ\text{C}$ m.p. $-40^\circ\text{C}$	$\text{PI}_3$ Dark red crystals m.p. $61^\circ\text{C}$ .	$\text{PCl}_5$ White crystalline solid, sublimes m.p. $148^\circ\text{C}$ b.p. $162^\circ\text{C}$	$\text{PBr}_5$ Yellow crystalline solid, decomposes on heating b.p. $106^\circ\text{C}$
Arsenic	...	$\text{AsCl}_3$ Colourless oily liquid b.p. $130.2^\circ\text{C}$ m.p. $-13^\circ\text{C}$	$\text{AsBr}_3$ Colourless crystalline solid m.p. $31^\circ\text{C}$ b.p. $221^\circ\text{C}$	$\text{AsI}_3$ Red hexagonal crystals m.p. $146^\circ\text{C}$	$\text{AsCl}_5$ Decomposes into $\text{AsCl}_3$ and $\text{Cl}_2$ above $-25^\circ\text{C}$	...
Antimony	...	$\text{SbCl}_3$ White crystalline solid. m.p. $73.2^\circ\text{C}$ b.p. $223.5^\circ\text{C}$	$\text{SbBr}_3$ White deliquescent needles m.p. $73^\circ\text{C}$ b.p. $280^\circ\text{C}$	$\text{SbI}_3$ Melting point of the stable form is $171^\circ\text{C}$	$\text{SbCl}_5$ Yellow mobile liquid b.p. $140^\circ\text{C}$	$\text{SbBr}_5$

TABLE I—(continued)

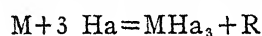
Compound	Trichloride	Tribromide	Triiodide	Pentachloride	Pentabromide
Element					
Bismuth	$\text{BiCl}_3$ White crystalline solid m.p. $227^\circ\text{C}$ b.p. $447^\circ\text{C}$	$\text{BiBr}_3$ Golden yellow crystals.	$\text{BiI}_3$ Black powder.	$\text{BiCl}_5$	$\text{BiBr}_5$

The question now arises as to whether the trihalides or the pentahalides can be regarded as saturated compounds. Let us first take as an illustrative example, the chlorides of phosphorus.

$\text{PCl}_3$  is diamagnetic, we may suppose that it has the constitution  $\text{P}^{+++}\text{Cl}^{-}$ . Each  $\text{Cl}^{-}$  ion is diamagnetic, and  $\text{P}^{+++}$  has the constitution  $1s^2 2s^2 2p^3 3s^2$ ; hence  $\text{P}^{+++}$  is also diamagnetic.

$\text{PCl}_5$  is also diamagnetic. We may suppose that it has the constitution  $\text{P}^{+5}\text{Cl}^{-}$ .  $\text{P}^{+5}$  has the inert gas constitution  $1s^2 2s^2 2p^6$ . If the above hypothesis be true, both  $\text{PCl}_3$  and  $\text{PCl}_5$  should show continuous absorption, as in the case of saturated halides of different valency. Chlorides of other elements in this group of periodic table will also behave likewise.

The calculation of the atomic heat of formation of  $\text{MHa}_3$  out of one M atom and 3 Ha atoms as expressed by the relation



is a matter of some difficulty; for usually the elements of this class, *viz.*, P, As, Sb and Bi vaporise in the polyatomic state. Preuner and Brockmüller<sup>1</sup>, in their extensive studies on the vapour pressure and vapour density of P and As find that at ordinary temperatures they vaporise as  $\text{P}_4$  and  $\text{As}_4$  which however break up into  $\text{P}_2$  and  $\text{As}_2$  as higher temperatures are reached. At still higher temperatures, the diatomic molecules may be broken up into atoms. The processes may be thus represented

$$[\text{P}] = \text{P}_4 + \lambda$$

$$\text{P}_4 = 2\text{P}_2 - \text{D}_1$$

$$\text{P}_2 = 2\text{P} - \text{D}_2$$

According to Preuner and Brockmüller<sup>1</sup>

$$\text{D}_1 = 31.5 \text{ K cal}; \text{D}_2 = 45.5 \text{ K cal} \text{ and } \lambda = 12.6 \times 124 = 15 \text{ K cal.}$$

Let  $S$  be the heat of vaporisation from the condensed state to the atomic state. Then we have

$$S = \frac{\lambda + D_1 + 2D_2}{4} = \frac{15 + 31.5 + 91}{4} = \frac{138}{4} = 34.5 \text{ K cal.}$$

Then

$$R = Q + S + \frac{3}{2} D_{HCl} - \lambda_{M_{HCl}}$$

In the case of  $PCl_3$

$$Q = 75.3 \text{ K cal.}; S = 43.5 \text{ K cal.}; \frac{3}{2} D_{HCl} = 86 \text{ K cal.}; \lambda_{M_{HCl}} = 9.3 \text{ K cal.}$$

$$R = 186.5 \text{ K cal}$$

The Table II has been compiled as shown below.

As regards Sb and Bi the dissociation has not been well studied. But probably they also vaporise as  $Sb_4$  and  $Bi_4$ . For Bismuth there is some indirect evidence. The absorption spectrum of Bi vapour has been studied by Barratt and Bonar<sup>2</sup> and has been found to consist of two indistinct systems of bands. One of them has been attributed to  $Bi_2$  molecule and from the convergence limit of this band the heat of dissociation of  $Bi_2$  molecule has been calculated amounting to 18.5 K cal. The other band system has been attributed to a molecule of greater complexity, probably  $Bi_4$ ; but unfortunately, it has not been analysed and we do not know the heat of dissociation of  $Bi_4$ .

Since all these molecules belong to the same group of periodic table we expect that the nature of their binding forces will be similar. As we know these values for a few of them we can get them for the others by interpolation. The interpolated values give at least the region where the correct values should lie. We take these values for the calculation of  $R$  in respective cases. The extrapolated values and their justification is to be seen from Tables 1, 2 and 3.

TABLE I

Reaction $M_2 \rightarrow 2M$	Heat of dissociation ( $D_2$ )	Difference
$P_2 \rightarrow 2P$	45.5 K cal	
$As_2 \rightarrow 2As$	38 K cal	7.5 K cal
$Sb_2 \rightarrow 2Sb$	(28) K cal	(10) K cal
$Bi_2 \rightarrow 2Bi$	18.5 K cal	(9.5) K cal

TABLE 2

Reaction $M_4 \rightarrow 2 M_2$	Heat of dissociation ( $D_1$ )	Difference
$P_4 \rightarrow 2 P_2$	31.5 K cal	6.5 K cal
$As_4 \rightarrow 2 As_2$	25 K cal	(9) K cal
$Sb_4 \rightarrow 2 Sb_2$	(16) K cal	(8.5) K cal
$Bi_4 \rightarrow 2 Bi_2$	(7.5) K cal	

TABLE 3

Substance	Heat of dissociation ( $D_2$ )	Heat of dissociation ( $D_1$ )	Difference
Phosphorus	45.5 K cal	31.5 K cal	14 K cal
Arsenic	38 K cal	25 K cal	13 K cal
Antimony	(28) K cal	(16) K cal	(12) K cal
Bismuth	18.5 K cal	(7.5) K cal	(11) K cal

TABLE II

N.B.—The values enclosed in brackets ( ) are uncertain because of the above interpolation.

Substance	Long wavelength limit " " in $^{\circ}A$	$Q_m = \frac{Nh\nu_m}{J}$ in K cal	Heat of reaction required to convert solid element into monatomic vapour in K cal per gm. atom "L"	Heat of formation of the salt in K cal per mole "Q"	Heat of vaporisation of the salt in K cal per mole "λ"	$\frac{R}{3}$ in K cal	$Q_m - \frac{R}{3}$ in K cal
Phosphorus Trichloride	2957	96.2	34.5	75.3	9.25	62.5	33.7
Arsenic Trichloride	5466	82.1	26.5	71.4	6.7	59.4	22.6
Antimony Trichloride	3256	87.3	(27.6)	91.4	11.05	(65.0)	(22.3)
Bismuth Trichloride	3656	77.2	(24.2)	90.6	18.3	(61.2)	(16.0)



## EXPERIMENTAL PROCEDURE

Of the four substances examined two were liquids and the other two were solids whose boiling points were less than  $500^{\circ}\text{C}$ . Two kinds of absorption vessels were used for producing the vapour and studying the absorption. For the solids (*i.e.*,  $\text{BiCl}_3$  and  $\text{SbCl}_3$ ) a pyrex glass furnace of the design shown in the diagram below was used (Fig. 1).

A is a pyrex glass tube of about one inch in diameter. BB are water-jackets employed for preventing the hot vapour from coming from the inside of the tube and depositing on the quartz windows C. The substances could be introduced in through the side tube D which also served for the connection to the vacuum pump. The furnace was heated by winding manganin wire round it and passing high electric current through it.

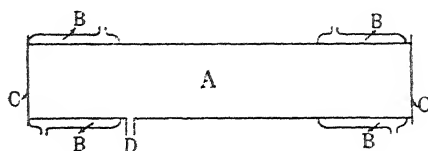


Fig. 1

For the liquid the furnace took the form of a long tube of glass of about an inch in diameter closed at its ends by quartz windows. Two side tubes provided with stop-cocks were attached to it. One of these tubes served for the connection to the vacuum pump. The other ended in a glass bulb of about 50c.cs. in capacity which was provided with a second stop cock for connecting the bulb to the atmosphere (Fig. 2).

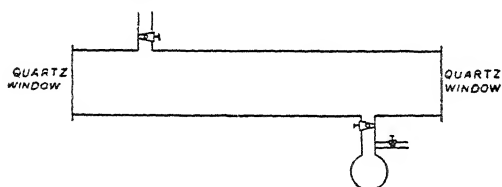


Fig. 2

The liquid under investigation, could be introduced into the side bulb. The vapour pressure in the long tube was varied by placing the bulb in baths of varying temperatures.

The vapour pressures could be measured in the first case by noting the temperature to which the furnace had been raised and taking from the tables (given in Mellor's 'Comprehensive Treatise in Inorganic and Physical Chemistry') the value of the saturation vapour pressures of the substance at that temperature. The temperature of the furnace was known by calibrating it in terms of the heating current when it had come to a steady state of temperature. In practice the steady state was brought about by heating it for at least ten minutes with constant current. Each photograph was taken only when the furnace had been heated by a definite electric current for about 30 minutes. In case of liquids the vapour pressure was measured by introducing a manometer on the pump side of the furnace.

The source for the continuous absorption was the Hydrogen tube run by a high current transformer at a current density of 100 M.A. The photographic plates used were Ilford Process plates.

#### DISCUSSION

We find (cf. Table II) that the energy of optical dissociation is in each of the compounds greater than the one-third of thermochemical value but less than two-thirds of it. Thus although one chlorine atom is shaken off from the molecule by the incident radiation, the energy expended is not strictly equal to that present in a single bond of the molecule.

Although a definite interpretation of these results cannot be offered unless more data accumulates, yet a few observations may well be made.

When a quantum of light, of energy  $h\nu$  ( $\nu$  being the long wavelength limit of the absorption spectrum) comes into contact with the molecule, it is absorbed by the latter. On its absorption it excites the electronic structure of the molecule which now corresponds to the state represented by a dipole  $MCl_2 - Cl^*$ . Here one electron passes over from the chlorine ion to the remaining portion of the molecule and we get dissociation product of the molecule, *i.e.*, a chlorine atom and a dichloride of the metal. It is found that the energy corresponding to the long wavelength limit of absorption is greater than the value for  $\frac{R}{3}$  in the case of all these chlorides. Franck in a recent note to the 'Naturwissenschaft' suggests that this difference is due to the vibrational energy given to the dichloride, which is obtained on photodissociation. The difference found in the present case is of the order of twenty to thirty K cal. This order of magnitude is too big to be due to vibrational changes only. It seems probable that this extra energy is involved in the electronic structure of the dichloride formed. The dichlorides are found as definite compounds, but unfortunately there does not exist any data about their heats of formation. This assumption is plausible as the electronic structure of the dichloride in the undissociated trichloride is not the same as that associated with the free dichloride. To bring about this change in the electronic structure some energy is obviously required. It manifests itself in the difference between  $\frac{R}{3}$  and  $Q_m$

The results may also be explained from a new hypothesis put forward by M. S. Desai of this laboratory (the hypothesis is yet unpublished for want of confirmatory experimental support). Desai considers the strength of each bond separately, making an assumption that if the fully developed compound be  $MX_n$  the unsaturated state  $MX_m$  ( $m < n$ ) will have a binding strength  $\frac{m}{n}$  times the fully developed compound. He calculates the strength

of each bond separately and adding them together equates it to the total thermo-chemical energy  $R$  and obtains the fraction which should give the beginning of absorption. He gets, for

$MX_4$	the limit of absorption to correspond to	$\frac{2}{3} R$
$MX_3$	" " " "	$\frac{1}{2} R$
$MX_2$	" " " "	$\frac{2}{3} R$
$MX$	" " " "	$R$

In the present case the absorption limit should be obtained by  $\frac{R}{2}$  and not  $\frac{R}{3}$  as given by Datta and Saha<sup>1</sup>. The agreement is seen from the following figures:—

TABLE 4

Substance	$Q_m$ (observed) K cal s	$\frac{R}{2}$ K cal s	$\frac{R}{3}$ K cal s
Phosphorous Trichloride ...	96.2	94.0	62.1
Arsenic Trichloride ...	82.1	86.5	57.4
Antimony Trichloride ...	87.3	(97.5)	(65.0)
Bismuth Trichloride ...	77.2	(92.2)	(61.2)

The agreement is tolerable in the case of  $PCl_3$  and  $AsCl_3$ . In the case of other two compounds nothing can be said either for or against Desai's hypothesis, as the heats of dissociation of  $Bi$ ,  $Sb$ , and  $Sb_2$  are not known. The value of  $R$  calculated by the help of the extrapolated values for those quantities cannot be accurate.

## ACKNOWLEDGMENTS

I have much pleasure in acknowledging my indebtedness to Prof. M. N. Saha, without whose extreme kindness on the author, this paper would not have seen the light of the day. My heartiest thanks are due to Prof. A. T. Mukerji of Patna for his having kindly lent me the use of the photomicrometer

and to Mr. Bholā Nath Ghosh research scholar Patna for rendering me material help during my stay there.

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- <sup>1</sup> G. Preuner and I. Brockmüller, *Zeit. f. Physik. Chem.*, 8, 129, 1913.
- <sup>2</sup> S. Barratt and A. R. Bonar, *Phil. Mag.* 9, 519, 1930.
- <sup>3</sup> A. K. Datta and M. N. Saha, *Bull. Acad. Sc. U. P.*, 1, 29, 1931-32.

# A NOTE ON THE VAPOUR PRESSURE OF ZINC BROMIDE

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Received April 3, 1933.

The purpose of this note is to correct a mistake that has been discovered after the publication of the paper on the same subject by the author,<sup>1</sup> in the last issue of the Bulletin.

In the said paper it is assumed that,

$$\begin{aligned} C - C_p &= 3R - \frac{5}{2}R \\ &= R/2 \end{aligned}$$

where  $C$  is the specific heat of the solid and  $C_p$  that of the vapour at constant pressure, but this is only true in the case of monatomic vapour. In the case of  $\text{Zn Br}_2$ , where the molecule is tri-atomic it requires modification.

$$C = 3R; \text{ and } C_p \text{ is given by } C_p - C_v = R$$

and

$$\begin{aligned} C_v &= C_{\text{rot}} + C_{\text{trans}} + C_{\text{osc.}} \\ &= \frac{3}{2}R + \frac{3}{2}R \quad (\text{neglecting the oscillation at low temp.}) \\ &= 3R \end{aligned}$$

$$\text{therefore } C_p = 3R + R = 4R$$

$$\begin{aligned} \text{and } C - C_p &= 3R - 4R \\ &= -R \end{aligned}$$

Thus Clausius' equation  $\lambda = RT^2 \frac{d \ln P}{dT}$  becomes,

$$- \ln P = - \frac{\lambda_0}{RT} - \frac{C - C_p}{R} \log T + k$$

$$\text{or } \log_{10} P = \frac{-\lambda_0}{2.3RT} + \log_{10} T + k'$$

This is of the form,

$$\log P = -\frac{A}{T} + \log T + B$$

Evaluating the constants A and B from the experimental data we have,

$$\left. \begin{array}{l} A = 5639.781 \\ \text{and } B = 9.066 \end{array} \right\} \dots \dots \dots (A)$$

Thus the vapour pressure equation for  $\text{Zn Br}_2$ , between  $300^\circ$  to  $400^\circ\text{C}$  becomes,

$$\log_{10} P = \frac{-5639.781}{T} + \log_{10} T + 9.066$$

And the latent heat of evaporation

$$\begin{aligned} \lambda_0 &= -4.554 A \\ &= 4.554 \times 5639.781 \\ &= 25.68 \text{ k cal} \dots \dots \dots (B) \\ &\text{instead of } 27.60 \text{ k cal as calculated before.} \end{aligned}$$

My thanks are due to Mr. B. N. Srivastava of our Department for pointing out the mistake.

#### *Reference.*

<sup>1</sup>M. S. Desai, *Bul. Acad. Sci.*, U. P., Allahabad, 2, 124, 1933.

ON THE TREMATODE PARASITES OF A RANGOON SILUROID  
FISH *CLARIAS BATRACHUS* (LINNAEUS 1785)

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In course of an investigation on the parasites of the common food fish of Rangoon, there were obtained from thirty *Clarias batrachus* (Linnæus 1785) a large number of trematodes which fall into three distinct groups. Two of these are members of the family Lepodermatidae Odhner 1911—one representing a new species and the other a new genus: the third is markedly different from the others and constitutes a new genus of a new sub-family belonging to the family *Acanthostomidae* Poche 1926.

***Astiotrema Spinosa* n. sp.**

These trematodes do not appear to be common: out of the thirty fish examined only three were found to be infected, with 35, 9, and 1 parasites respectively. In hosts one and three the parasites were obtained from the intestine and in host two from the posterior part of the stomach. Body when

contracted, pegshaped, with a much broader anterior end: when elongated margins more or less parallel, with rounded anterior and posterior ends. In the extended condition the anterior end is greatly narrower ending in a blunt point. Body musculature more strongly developed between suckers, so that, when the animal contracts, the two suckers lie very close to each other. Cuticle covered with sharp pointed spines arranged in transverse rows, the spines of each row alternating with those of the preceding and succeeding rows. Spines become progressively denser on the surface from the posterior to the anterior part of the body. Body length 1.3\*—3, maximum breadth 0.4 0.58, with a much smaller variation than in the length. Suckers powerful. Ventral sucker in first third of the body, circular in outline, approximately 0.185 in diameter. Oral sucker, slightly smaller than ventral, 0.18 × 0.16. Genital opening closely anterior to ventral sucker. Small prepharynx present. Pharynx spherical, 0.08 in diameter in extended specimen. Oesophagus moderately long, from 1—2½ times length of pharynx. In rare cases, *i.e.*, in fully extended condition its length extends to four times the length of pharynx. Intestinal bifurcation between the two suckers, its position changing with the degree of expansion. Intestinal caeca more or less straight, terminating a little anteriorly to posterior end of body.

Testes rounded, large, usually equal, 0.16—0.28 in diameter, situated obliquely one behind the other in posterior half of body just behind the middle, occupying most of the space between the caeca. In a few preserved and mounted specimens, however, the testes were found to be elliptical and broader than long. Cirrus sac large, extending much behind the ventral sucker as far as ovary, 0.23—0.44 in length, posteriorly saccular, almost entirely filled by the large vesicula seminalis and continuing anteriorly into a tubular structure. Parsprostatica tubular and long separated from vesicula seminals by constriction. Ductus ejaculatorius small, continuing into a small cirrus. Male opening shallow, genital atrium to the right of the female opening.

Ovary spherical, 0.13—0.17 in diameter, approximately in median line, much posterior to ventral sucker except in contracted specimens. Receptaculum seminis large, pearshaped and elongated transversely, joining oviduct near ovary. Coils of uterus rarely overlapping gut diverticula except in the part posterior to testes. Vitellaria composed of numerous follicles, scattered without any definite arrangement along lateral margins of body overlapping intestinal caeca from posterior margin of ventral sucker to anterior margin of posterior testis or sometimes to posterior margin of the latter, in which case the distance between the two testes was considerably less than normal and that from the posterior testis to posterior end of the body correspondingly

\* All measurements are in millimetres.



increased, indicating an anterior displacement of that testis. The follicles greatly extend inwards to the median line in the region between ovary and anterior testis. Eggs numerous, with a yellowish-brown shell,  $0.027 \times 0.0114$ .

The present species has a small protrusible cirrus and a large cirrus sac extending much behind the ventral sucker which is characteristic of the genus *Astiotrema*. It thus agrees with the definitions given by various investigators (Looss, Odhner, and Mehra). The Burmese form differs from *Astiotrema loossii* Mehra 1930 in the relative size of its suckers, in the absence of lobed margins to the testes and ovary, and in the extent of vitellaria and intestinal caeca. From *A. impletum* Looss 1899, *A. monticellii* Stossich 1904 and *A. elongatum* Mehra 1930 it is separated by a difference in the position of the intestinal fork and by the extent of intestinal caeca and vitellaria. It is closer to *A. reniferum* Looss 1899 but differs from that species in the extent of the intestinal caeca and vitellaria. These differences necessitate the creation of a new species, for which the name *Astiotrema spinosa* is proposed. This is the second species of the genus *Astiotrema* obtained from fish, the first *Astiotrema impletum* Looss 1899, being recorded from *Tetradon fahaka*.

#### ***Ganada clariae* n. gen., n. sp.**

This species is common: out of the fish examined nineteen were infected, the number of parasites from each fish ranging from 1.63, with an average of 10–15. Body, when contracted, oval, with much broader anterior end: when extended, cylindrical, with rounded ends. Length 1.53–2.8 maximum breadth 0.3–0.4. Cuticle with small spines, progressively denser from posterior to anterior part of body. Suckers powerful, ventral slightly larger than oral,  $0.133$ – $0.18$  and  $0.114$ – $0.164$  in diameter respectively. Genital pore immediately in front of ventral sucker to the left of the median line and some distance behind intestinal bifurcation. Prepharynx short,  $0.04$ – $0.046$  in length. Pharynx  $0.08$ – $0.14$  in diameter. Oesophagus  $0.027$ – $0.061$  in length. Intestinal caeca wide, comparatively broad anteriorly and extending almost to posterior end of body.

Testes median, in the third quarter of the body, the distance separating the two differing in different specimens. Posterior testis slightly bigger than anterior, approximately  $0.152$ – $0.21$  and  $0.137$ – $0.18$  in diameter respectively. Genital atrium small,  $0.02$ – $0.04$  in diameter. Cirrus sac somewhat tubular, slightly semilunar,  $0.234$ – $0.4$  in length, dorsal to ventral sucker extending slightly posteriorly to it. Vesicula seminalis divided into internal and external portions, the latter lying close to ovary. Pars prostatica small, near anterior margin of ventral sucker. Cirrus small. Male opening to the right side of the female.

Ovary approximately spherical,  $0.114$ – $0.162$  in diameter, lying in middle third of body. Vitellaria composed of numerous, closely crowded big follicles

long lateral margins of body, more densely crowded in post-testicular region than in pre-testicular. Uterus much coiled, the loops running transversely and passing between testes. Eggs numerous, thin-shelled, approximately  $0.018 \times 0.012$ . Excretory bladder Y-shaped and typical of the family Lepodermatidae Looss 1899.

The parasite agrees with the characters of the subfamily Lepodermatinae Looss 1899, but differs from all the genera included in it in the presence of an external vesicula seminalis. In the absence of a receptaculum seminis it resembles *Lepoderma* Looss 1899 and *Haplometra* Looss 1899 but differs from the other genera. The tandem position of the testes and the position of the cirrus sac further separate it from *Lepoderma* Looss 1899. From *Haplometra* Looss 1899 it differs in the rudimentary nature of its oesophagus and the pronounced coiling of the uterus in addition to the presence of its characteristic external vesicula seminalis. By its possession of the latter organ it exhibits a certain affinity with the genus *Leptophallus* Luhe 1909, but this similarity does not extend to other characters. A new genus is therefore created for the reception of these parasites, for which is proposed the name *Ganada* with *Ganada clariae* as the type genus and species.

#### Diagnosis of the genus *Ganada* n. gen.

Lepodermatidae: Lepodermatinae. Cuticle with spines. Suckers almost spherical, unequal, Prepharynx and pharynx present. Oesophagus very short. Genital pore left of the median line, anterior to ventral sucker. Intestinal caeca extending to hinder part of body. Testes post-ovarian and median, the posterior larger than the anterior. Ovary smaller than testes. Cirrus sac semilunar and median, extending dorsally over ventral sucker. Vesicula seminalis divided into internal and external vesiculæ seminales. Receptaculum seminis absent. Vitellaria with closely packed follicles, extending from behind ventral sucker to posterior end of body. Uterus much coiled, nearly reaching posterior end of body and containing numerous eggs. Excretory bladder Y-shaped and typical of the sub-family.

#### \**Masenia collata* n. gen., n. sp.

On examination of thirty fish, eighteen were found infected with these parasites, in the intestine or posterior part of the stomach, the number in each case ranging from 1-390.

\* From *masen*, the Chingpo for spine or thorn.

Body club-shaped, 0.7—1 in. length with a maximum breadth of 0.24—0.36. Cuticle for anterior three-fourths of body armed with small sharp spines which become progressively denser from the posterior to the anterior part of the body. In addition to body spines, approximately 53 oral spines are present around the oral sucker, arranged in two regular rows—one above the other. Suckers very powerful. Oral larger than ventral, 0.12—0.17 × 0.11—0.15, funnel-shaped, extending for a considerable distance within the body. Ventral sucker circular, 0.1—0.13 in diameter, lying in anterior half of body. Prepharynx short, approximately 0.02—0.045 in length. Pharynx 0.04—0.54 × 0.025—0.042. Oesophagus small, dividing immediately anteriorly to ventral sucker into two wide caeca which, maintaining approximately the same width throughout, extend to the level of the posterior testis.

Testes approximately equal and spherical, 0.07—0.1 in diameter, lying close to each other in middle third of body. Genital atrium in form of a shallow depression close behind oral sucker. Cirrus sac large, of greater length than half the length of the body, divisible into two distinct portions, a broad basal part and a long narrow coiled tubular structure, the latter fitting into the neck of the former. Vesicula seminalis mostly within saccular part of cirrus sac, divided by a constriction into a smaller proximal and a large distal portion, and extending for a short distance in its tubular part. Pars prostatica and ductus ejaculatorius enclosed within narrow tubular part of cirrus sac. (In Fig. 5 not visible being hidden by gland cells). Cirrus small and in most of the fixed specimens distinctly protrudes out of the body in region of oral sucker.

Ovary immediately behind ventral sucker, approximately spherical, 0.064—0.12 in diameter, slightly bigger than testes. Receptaculum seminis large, posterior to ovary, often obscured by the large number of eggs in its vicinity. Vitellaria composed of follicles extending from anterior margin of ventral sucker to middle of posterior testis. Uterus voluminous, the convolutions occupying practically the whole body posterior to intestinal bifurcation and too closely crowded for the individual coils to be observed. Passing backwards from its origin behind the ventral sucker it forms a double sinuous course in the post-acetabular region. It then turns forward and, still with a slightly sinuous course, opens into the genital atrium immediately behind the oral sucker. Eggs numerous, oval, with a yellowish shell approximately 0.02 × 0.012. Excretory bladder broad and tubular, extending anteriorly to posterior testis; cornua not visible probably owing to the crowding of genital glands, cirrus sac and ventral sucker.

The peculiar oral spines and the funnel-shaped oral sucker of *Masenia collata* n. sp. are characteristic of some of the genera of the family *Acanthostomidae* Poche 1926 (Syn. *Acanthochoasmidae* Nicoll (1914) to which it exhibits closer relationship than to any other family. The presence of a well-developed

cirrus sac in *Masenia*, however, separates it from all the genera of the *Acanthostomidae* Poche 1926 except those included in the sub-family *Anoictostominae* Nicoll 1914. The affinities, however, are very close with *Anoictosoma planicolle* Rud. 1819) as exhibited in the following characters:—

1. Large funnel-shaped oral sucker.
2. Presence of oral spines.
3. Size of suckers.
4. Well-developed cirrus sac enclosing large vesicula seminalis.
5. Ovary close behind ventral sucker.
6. Uterine coils mainly posterior to testes.

On the other hand, the differences between them are so great that it is not possible to include *Anoictosoma planicolle* (Rud. 1819) and my species in the same genus. In most of the genera of the family *Acanthostomidae* Poche 1926, the cirrus sac has been recorded to be absent and in some cases where it is present as in *Anoictosomum* Stossich 1899 it does not extend much in front of the anterior margin of ventral sucker. *Masenia* n. gen. has a long tubular coiled cirrus sac which remarkably differs in shape and size from that of all the genera of the family *Acanthostomidae* Poche 1926. Further, the genital opening in this family lies immediately anterior to ventral sucker but in *Masenia* n. gen. it has become shifted much forward and occupies a position dorsal to oral sucker. It is therefore considered necessary to create a new genus *Masenia* with *M. collata* as the type species. Though the genus shows closer relationship with the sub-family *Anoictostominae* Nicoll 1914 than *Acanthostominae* there does not seem to be any doubt that it belongs to a new sub-family *Maseniinae* on account of two important characters, i.e., the form and size of the cirrus sac and the position of the genital pore.

Key to the sub-families of the family *Acanthostomidae* Poche 1926:—

1. Cirrus sac absent ... *Acanthostominae*.
- Cirrus sac present ... 2.
2. Genital opening close in front of ventral sucker ... *Anoictostominae* Nicoll 1914.

Genital opening far in front of ventral

sucker and near oral sucker... *Maseniinae* n. sub. fam.

Diagnosis of the sub-family *Maseniinae* n. sub. fam.:—

*Acanthostomidae*: Body small. Cuticle spinose. Suckers strongly developed: oral sucker funnel-shaped, much larger than ventral, with strong spines arranged in two rows. Prepharynx present. Pharynx well developed. Oesophagus of moderate length. Cirrus pouch long and coiled, divisible into a basal saccular part and a distal tubular portion nearly reaching anterior end of body. Vesicula seminalis large, its basal portion saccular. Cirrus small and unarmed. Testes post-acetabular, post-ovarian, nearly spherical. Re-

receptaculum seminis present. Vitelline glands with large follicles extending between ventral sucker and posterior testis. Uterus very voluminous, convoluted, occupying greatly hinder part of the body behind testes. Excretory bladder tubular, extending to posterior testis. Eggs numerous and oval. Parasites of siluroid fish (*Clarias batrachus*).

Type genus *Masenia* n. gen.

Type-species *M. collata* n. gen., n. sp.

#### ACKNOWLEDGMENT

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#### EXPLANATION OF PLATES

Fig. 1. *Astiotrema spinosa*—Ventral aspect (camera lucida drawing).

Fig. 2. *Ganada clariae* n. gen., n. sp.—Ventral aspect (camera lucida drawing).

Fig. 3. *Ganada clariae* n. gen., n. sp.—Horizontal-Longitudinal section.

Fig. 4. *Masenia collata* n. gen., n. sp.—Ventral aspect (camera lucida drawing).

Fig. 5. *Masenia collata* n. gen., n. sp.—anterior part of the body showing cirrus sac.

#### LETTERING TO FIGS. 1—5

c. cirrus, c. s. cirrus sac, e. s. v. vesicula seminalis externa, g. o. genital opening, i. c. intestinal caecum, i. s. v. vesicula seminalis interna, oes. oesophagus, o. s. oral sucker, ov. ovary, p. g. prostate gland cells, ph. pharynx, r. s. receptaculum seminis, s. g. shell gland, s. v. vesicula seminalis, t.<sub>1</sub>. anterior testis, t.<sub>2</sub>. posterior testis, ut. uterus, vit. vitellaria, v. s. ventral sucker.

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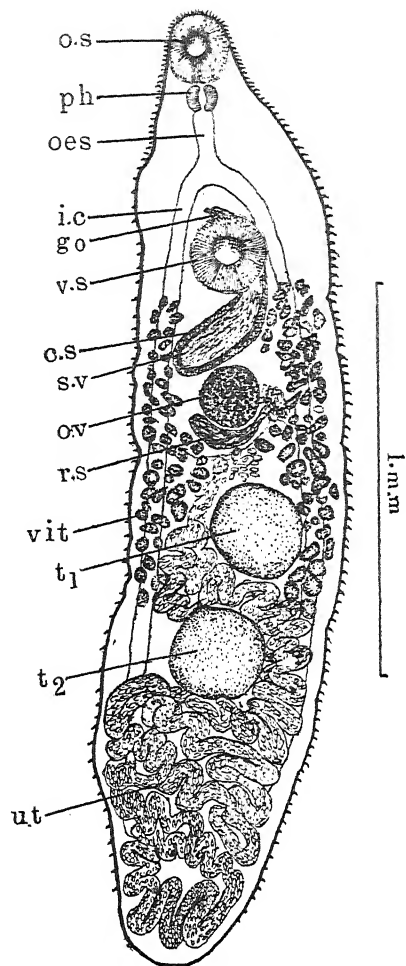


FIG 1.—*Astiotrema spinosa*, n. sp., ventral aspect.

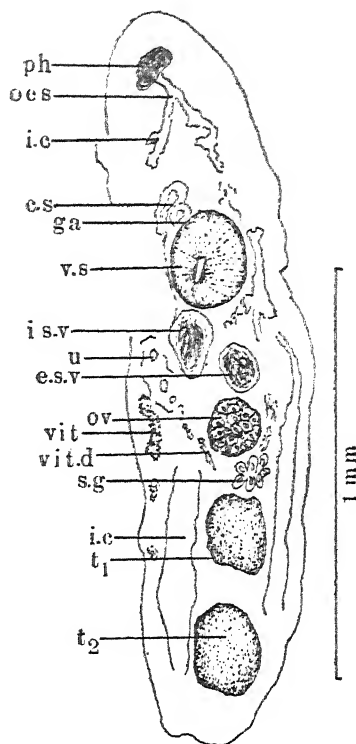


FIG 3.—*Ganada Clariae*, n. gen., n. sp., horizontal-longitudinal section

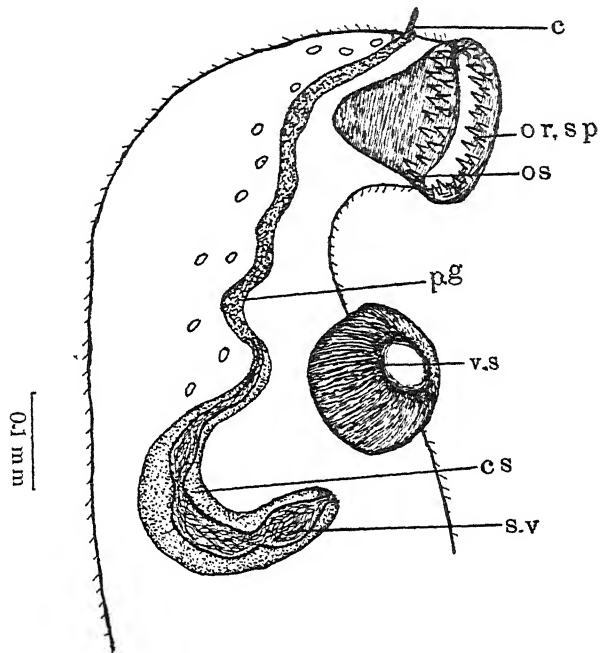


FIG 5.—*Masenia collata*, n. gen., n. sp., anterior part of the body showing cirrus sac.

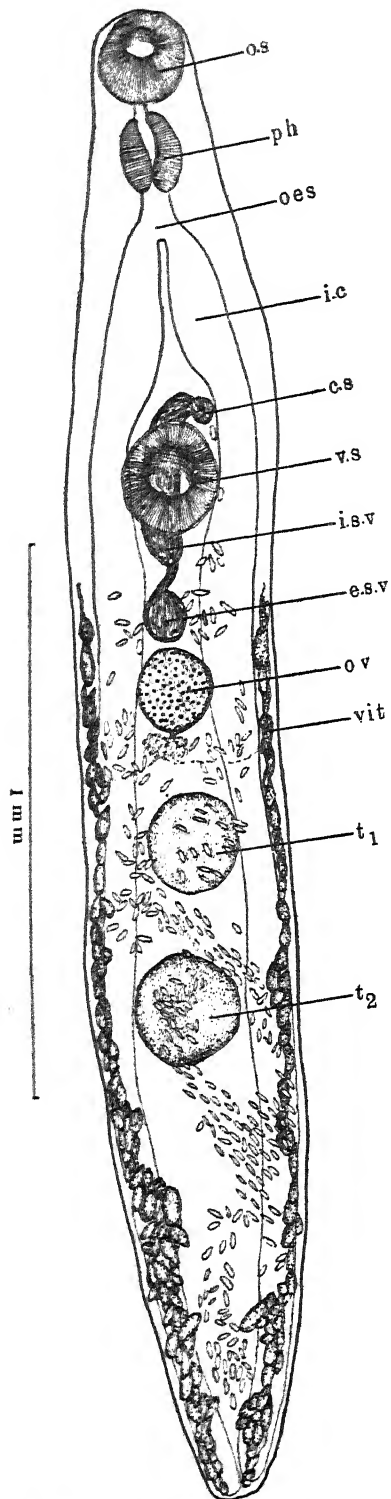


FIG 2.—*Ganada clariae*, n. gen., n. sp., ventral aspect.

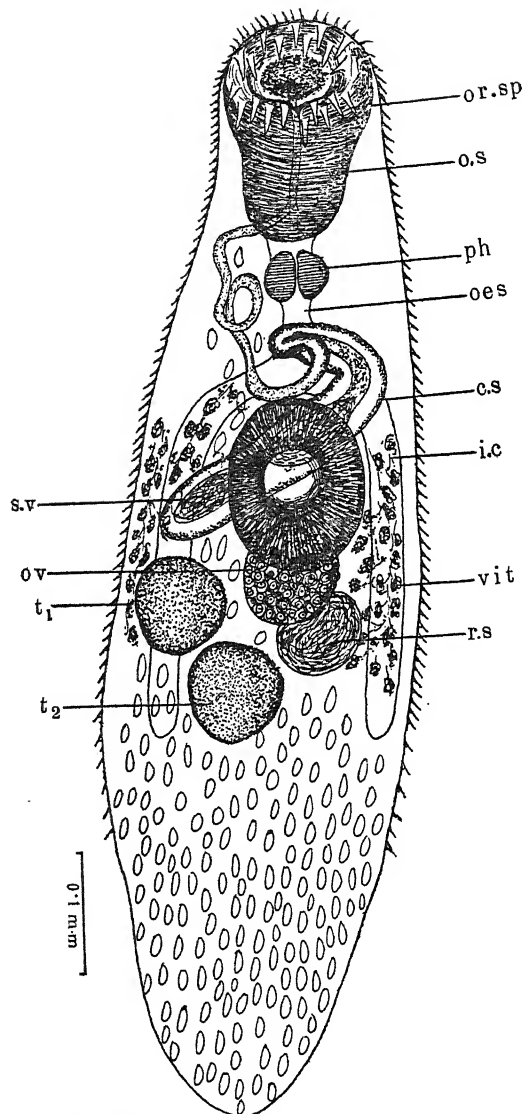


FIG 4.—*Masenia collata*, n. gen., n. sp., ventral aspect.



# ON NEW TREMATODES OF FROGS AND FISHES OF THE UNITED PROVINCES, INDIA.

**Part I.**—New Distomes of the family Hemiuridæ Luhe 1901 from North Indian fishes and frogs with a systematic discussion on the family Halipegidæ Poche 1925 and the genera Vitellotrema Guberlet 1928 and Genarchopsis Ozaki 1925.

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## Introduction

In this paper are described a number of new distomes obtained from the gut of the frogs and fishes of Sitapur, Lucknow and Allahabad. Besides these forms a large number of other new species were obtained, which I shall describe in subsequent papers. The account of all these trematodes forms the subject of a thesis which I submitted in March 1933 in partial fulfilment of the requirements for the degree of Master of Science of the Allahabad University, and which for the purpose of publication I have split up into four parts. The first part as contained in this paper deals with two new species and a new variety of the genus *Halipegus* Looss 1899, two new species of the genus *Progonus* Looss 1899 and two new species of the new genus *Ophiocorchis*.

The systematic position of the family *Halipegidae* Poche 1925 and subfamily *Derogenetinae* Odhner 1927 of the family *Hemiuridae* Luhe 1901 is fully discussed and the author comes to the conclusion that the family *Halipegidae* is untenable and that the genus *Halipegus* should be included in the subfamily *Derogenetinae* of the family *Hemiuridae*. The synonymy of the genera *Vitellotrema* Guberlet 1928 and *Halipegus* Looss 1899 is clearly indicated. In the light of the new forms belonging to the genus *Progonus* which are described in this paper the genus *Genarchopsis* Ozaki 1925 is dropped and the species belonging to it is referred to the genus *Progonus*.

I am deeply grateful to Dr. H. R. Mehra, under whose supervision it has been my proud privilege to work, for his valuable suggestions and helpful criticisms and his constant and sympathetic interest in the preparation of this work. I am also indebted to him for the free use of his extensive private library and for translating several papers written in foreign languages. I am also indebted to Dr. D. R. Bhattacharya for kindly providing me laboratory facilities during holidays. I thank Mr. S. C. Verma for the general interest he has taken in this work.

Genus *Halipegus* Looss 1899

Vulpian in 1860 described *Distomum ovocaudatum*, parasitic in the mouth cavity and pharynx of European frogs. In 1872 Grebnitzky published an account of a new species of *Distomum* *Dist. kessleri* from the stomach of *Rana esculenta*. His paper was published in a rather obscure journal and therefore the parasite remained unknown till recently (1929). In 1890 Creutzburg worked the life history of *Dist. ovocaudatum*. This distome was also described independently by Sonsino and Looss in 1894. Five years later Looss created the genus *Halipegus* for *Dist. ovocaudatum*, assigning it somewhere near the *Syncoelinae*. In 1905 Stafford recognized *Dist. ovocaudatum* Nickerson 1898, parasitic in *Rana calasbiana* and *R. clamata* to be a new species which he called *Halipegus occidualis*. Ssinitzin who in 1905 worked out the life cycle of *Cercaria cystophora* described by Wagener in 1866 from *Planorbis marginata* found that the cyst passes into a dragon-fly larva—*Colopteryx virgo*, which serves as its intermediate host and the frogs feeding on these larvæ become infected with *H. ovocaudatum*. Poche in 1925 created a new family *Halipegidae* for the genus *Halipegus* and placed it under his superfamily *Hemiuroidea* Poche 1925. As will be seen from the systematic discussion the family *Halipegidae* is untenable. Guberlet in 1928 added a new genus *Vitellotrema* from the stomach of a water snake to the family *Halipegidae* Poche. This genus which is based only on the unlobed character of the vitellaria cannot be maintained and I accordingly assign it to the genus *Halipegus* Looss as *H. fusipora*. In the following year Isaitschikow described *H. rossicus* from the stomach of a Russian frog—*R. esculenta* and Bychowsky referred *Dist. kessleri* Grebnitzky to the genus *Halipegus*. Wlassenko in 1929 found specimens of *H. kessleri* in the stomach of *Natrix natrix* pointing out the close resemblance between them and *H. rossicus*. The latter form he regarded as synonymous with *H. kessleri*. Simer found in the intestine of *Polyodon spathula* *H. perplexus*, which he described in 1929. I add in this paper to the genus *Halipegus* two new species and a new variety from the stomach and intestine of Indian frogs.

***Halipegus mehransis* n. sp.**

(Fig. 1)

Host—*Rana cyanophlyctis*.

Habitat—Stomach.

Locality—Sitapur, Oudh (India).

This is the most common trematode infecting *R. cyanophlyctis* in Sitapur district from May to September. During this period the frogs were found to be invariably infected with this parasite. The average intensity of infection is about 4 although the number varies from 2—12 in a single host. The distomes live firmly attached to the wall of the stomach by means of their large

and powerful ventral sucker which, when in use, may be prominently protruded from the body. The free anterior end may be extended for some distance in a leech-like manner. The parasites have little power of adaptability to changed conditions of diet and temperature, for none could live for more than 28 hours in nutritive solutions kept at the laboratory temperature (*vide* Table I).

Table I

Date 2nd—6th August, 1932. Laboratory Temp. 70—82.5°F.

Nutritive solutions used.	Physiological salt solution 0.75%	Phys. salt sol. and yolk 1:2	Phys. salt sol. and albumen 1:1	Phys. salt sol. and yolk and albumen 1:2:2	5% sugar solution.
Number of parasites kept ...	5	5	5	5	5
Maximum number of hours they lived ...	6	20	18	28	8½
Number of parasites lived for maximum period ...	2	3	2	3	1

The parasites are light grey in colour and have enormous power of contraction and expansion. The thick and muscular body is spindle-shaped with bluntly pointed ends and a nearly circular cross-section. The bodywall is smooth and entirely devoid of spines. It may be thrown into circular folds which are more prominent with increased degree of contraction. These folds give the trematodes a ringed appearance in the living condition. The sexually mature worms when alive vary from 1.7—4.5 mm in length and 0.6—0.9 mm in breadth. In entire mounts the parasites measure 3.1—5.1 mm in length and 0.9—1.2 mm in maximum breadth which lies about the region of the acetabulum.

The suckers are well developed and highly muscular and have a circular outline. The subterminal and ventrally directed oral sucker of 0.28 mm diameter is nearly half the size of the acetabulum. The acetabulum of 0.5—0.72 mm diameter and 1.8 times the size of the oral sucker, situated in the middle of the body, is very deep extending nearly to the dorsal surface of the body.

The genital pore is ventral, usually median, rarely shifted slightly to one side; a little behind the intestinal bifurcation. The excretory pore lies terminally at the extreme hinder end of the body.

The pharynx, 0.1—0.14 mm in diameter, is situated just posterior and slightly dorsal to the oral sucker. The oesophagus being absent the intestinal caeca arise directly behind the pharynx. The caeca are broad and wavy with several marked constrictions and extend up to the extreme posterior end.

The massive testes are extracaecal and have roughly triangular outline. They are situated obliquely, one on each side, close behind the anterior half of the body. The right testis,  $0.3-0.6 \times 0.27-0.43$  mm in size, lies in the space between the right intestinal caecum and the bodywall, with its major portion in level with the posterior third part of the acetabulum. The left testis of  $0.3-0.6 \times 0.28-0.4$  mm size is situated  $0.09-0.17$  mm behind the acetabulum in the space between the left intestinal caecum and the bodywall. The vesicula seminalis is a flask-shaped structure of  $0.22-0.25 \times 0.12-0.14$  mm size, situated slightly to the right of the median line, a little behind the intestinal bifurcation. The vesicula seminalis narrows anteriorly to form the ductus ejaculatorius,  $0.08 \times 0.01$  mm size, which bends downwards on the left side of the vesicula seminalis to open on a nipple-shaped cone or papilla lying in the genital atrium. The ductus ejaculatorius is surrounded by prostate gland cells which lie free in the parenchyma.

The ovary is nearly spherical in shape with entire margin, measuring  $0.16-0.27 \times 0.2-0.3$  mm in size. It is intracaecal lying just in front of the left vitelline gland close to the median line. The oviduct arising from the middle of the posterior margin of the ovary turns towards the median line and after running for a short distance is joined first by the Laurer's canal and soon after by the common vitelline duct, before it enters the compact shell gland mass of an oblong form and  $0.17-0.25 \times 0.11-0.17$  mm size, situated obliquely to the right side in level with the ovary. The shell gland mass is separated from the latter by the anteriorly passing Laurer's canal. The Laurer's canal of  $0.03$  mm diameter has a number of transverse constrictions at regular intervals, throughout its length. A receptaculum seminis is absent. The relations of the female ducts are shown in figure 8.

The vitellaria lie in two groups, ventral to the intestinal caeca, one on either side of the median line close behind the ovary and the shell gland mass, ending posteriorly a little in front of the blind extremities of the intestinal caeca. The right vitelline gland,  $0.3-0.46 \times 0.2$  mm in size, consists of four well marked lobes while the left one,  $0.27-0.35 \times 0.18-0.2$  mm in size, has five lobes. The vitelline duct of each side runs mesially and the two ducts meet in the median line to form the common vitelline duct just behind the shell gland mass.

The initial part of the uterus is filled with a huge number of sperms and hence may be regarded as receptaculum seminis uterinum. The uterus forms closely packed and irregularly arranged transverse convolutions which extend laterally up to the bodywall both in front and behind the acetabulum. The uterine coils, however, never extend behind the shell gland mass and the vitellaria. In its terminal part the uterus lies parallel to the vesicula seminalis and like the ductus ejaculatorius is lined internally with cuticle. The uterus opens on the genital papilla very close to the male opening. It contains numerous eggs of golden yellow colour which bear a very long filament

at one end. The egg measures  $0.045 \times 0.018$  mm in size and filament 0.32 mm in length. The filament is 7 or 8 times the length of the egg.

The excretory bladder is Y-shaped, consisting of an unpaired excretory vesicle extending from the posterior end right up to the level of the testes where it divides into two lateral cornua. The cornua extend anteriorly, one on either side, uniting with each other on the dorsal side of the pharynx.

This species bears a very close resemblance to *H. occidualis* Stafford in the position of suckers, the extent of the intestinal caeca, lobed nature of the vitellaria, position of the gonads and the excretory pore. The important differences which mark it out as a new species are: the absence of the oesophagus, position of vitellaria and the genital pore, the union of the cornua of the excretory bladder in the region of the pharynx and not above the oral sucker, the size of the ova and the length of their filaments which are 7 or 8 times as long as the ovum and not shorter than the latter as in *H. occidualis*.

***Halipegus mehransis* var. *minutum* n. var.**

(Fig. 2)

Host—*Rana tigrina*.

Habitat—Stomach.

Locality—Sitapur, Oudh (India).

Eight specimens of this parasite were obtained from the stomach of two out of about sixty frogs examined during the rainy season in 1932. The distomes have a muscular and cylindrical body with marked power of contraction and expansion. In the living condition the parasites measure 0.9–1.9 mm in length and 0.36–0.7 mm in maximum breadth. Sexually mature worms in entire mounts measure 1.6–1.9 mm in length and 0.6–0.7 mm in maximum breadth across the testicular region.

The subterminal and ventrally directed oral sucker is transversely oval in outline, measuring  $0.14-0.2 \times 0.19-0.27$  mm in size. The acetabulum, situated about the end of the anterior half of the body, is spherical in outline with a diameter of 0.3–0.4 mm. It is one and a half times as large as the oral sucker. The pharynx is spherical with a diameter of 0.08–0.1 mm and opens directly into the two broad and wavy intestinal caeca which extend up to the extreme posterior end of the body.

The topography of the gonads and the structure of the copulatory apparatus are very much similar to those of *H. mehransis* n. sp. The testes, ovary, vitellaria and the shell gland complex all occupy the same relative positions as in the above species. The testes are extracaecal, lying somewhat asymmetrically one on each side, close behind on the sides of the acetabulum. The left testis,  $0.16-0.26 \times 0.11-0.17$  mm in size, lies a little cephalad of the right testis which measures  $0.12-0.25 \times 0.11-0.19$  mm in size. The vesicula seminalis of  $0.14-0.17 \times 0.08-0.096$  mm size is situated slightly to the right of the median line close behind the intestinal bifurcation. The

short ductus ejaculatorius of about 0.075 mm length is surrounded by a few prostate gland cells. The male pore lies on a small conical papilla situated in the genital atrium.

The ovary is a small spherical structure of 0.10–0.11 × 0.1–0.14 mm size, situated close to the right side of the median line just in front of the right vitelline gland. The shell gland complex, 0.096–0.14 × 0.06–0.08 mm in size, lies obliquely behind the ovary on the side opposite to that of the ovary just in front of the left vitelline gland. A conspicuous Laurer's canal is present but the receptaculum seminis is absent.

The vitellaria lie in two somewhat obliquely situated groups, one on each side close behind the ovary and the shell gland complex. The right vitelline gland of 0.14–0.22 × 0.096–0.19 mm size has five distinct lobes, while the left gland, 0.16–0.22 × 0.06–0.14 mm size, has only four such lobes. The vitelline ducts of both sides unite to form a common duct in the median line just behind the ootype.

The first crop of eggs produced are abortive. Two specimens of 1.6 mm length had the earliest eggs still in the uterine convolutions in the post-acetabular region. The most anterior eggs were about half the normal size and had irregular shape with very thin shell. The contents of the eggs were vacuolar in appearance. Fully mature specimens, however, have numerous golden yellow eggs in the transversely arranged coils of their uterus. The uterine convolutions are confined to the intercaecal area and never extend posterior to the ovary and the shell gland complex. Terminally the uterus opens on the genital papilla independently of the male opening. The eggs of 0.042–0.047 × 0.02 mm size have at one end a very long filament of 0.17–0.2 mm length. The excretory system is as in *H. mehransis*.

This trematode shows very close resemblance to *H. mehransis* in the general form, shape and the topography of the various organs, but differs from it in the smaller size of the body and of the various organs, transversely oval shape of the oral sucker, position and size ratio of the acetabulum and the size of the egg and its filament and the host. On the basis of these differences I consider this parasite to belong to a new variety of *H. mehransis* which I name *var minutum* on account of its much smaller size.

### ***Halipegus spindale* n. sp.**

(Fig. 3)

Host – *Rana cyanophlyctis*.

Habitat – Intestine

Locality – Sitapur, Oudh (India).

Four mature specimens of this parasite were obtained from the intestine of a frog examined in July 1932 at Sitapur. The parasites have a smooth and spindle-shaped body nearly circular in cross-section. The specimens fixed

under a slight pressure measure 3.1—3.4 mm in length and 0.86—0.96 mm in greatest breadth which lies across the acetabular region. The genital atrium is situated a little behind the intestinal bifurcation slightly to the left of the median line. The excretory pore lies subterminally on the ventral surface near the hinder end of the body.

The suckers are well developed, muscular and have a circular outline. The oral sucker, of 0.26 mm diameter, is situated subterminally on the ventral surface. The acetabulum, 0.52 mm in diameter, is situated in the region between the anterior third and the first half of the body. The acetabulum is twice as large as the oral sucker.

The oral sucker leads posteriorly into a thick-walled spherical pharynx of 0.1—0.11 mm diameter which is followed by a very short oesophagus of 0.05—0.08 mm length. The intestinal caeca, with markedly crenated outline, are of uniform breadth, extending more or less in a straight course up to the anterior end of the vitellaria.

The testes are more or less ovoid in shape and lie asymmetrically outside the intestinal caeca, close behind the acetabulum. The left testis, 0.38—0.41 × 0.27—0.32 mm in size, is more cephalad, lying in part to the side of the acetabulum, while the right testis, 0.38—0.4 mm in size, is situated more caudad at about 0.17—0.2 mm distance behind the acetabulum. The vesicula seminalis of 0.21—0.22 × 0.1—0.12 mm size is a prominent bulb-shaped structure situated behind the intestinal bifurcation a little to the left of the median line. It is continued into a short bent ductus ejaculatorius of 0.13 mm length which opens on a short conical genital papilla situated in the shallow genital atrium. A number of prostate gland cells lie all round the ductus ejaculatorius.

The ovary, 0.16—0.17 × 0.19—0.22 mm in size, is intracaecal, situated to the right side in front of the right vitelline gland. The shell gland mass is an oval structure of 0.12 × 0.16 mm size, lying either in the median line or slightly to the left side close in front of the vitelline gland of the same side. The relations of the female genital ducts are as in *H. mehransis*.

The vitellaria lie in two groups, one on either side, at the extreme hinder end of the body and immediately behind the blind extremities of the intestinal caeca. The right vitelline gland of 0.3—0.32 × 0.24 mm size consists of five lobes while the left one measuring 0.3—0.32 × 0.16 mm in size has only four such lobes.

The uterus which is enormously developed and stuffed with numerous golden yellow eggs lies mostly in transverse convolutions extending up to the bodywall on either side. Terminally it opens on the small genital papilla situated in the shallow genital atrium. The eggs measure 0.045—0.047 × 0.02—0.022 mm in size and bear at their posterior end a long filament of 0.27 mm length.

The excretory system is as in *H. mehransis* with the difference that in this form the excretory pore lies subterminally on the ventral surface.

Of all the species of the genus *Halipegus* *spindale* bears a close relationship to *H. mehransis* in the form of the body, position of gonads and vitellaria, the relations of the female genital ducts and in the structure of the end apparatus of the reproductive organs. It differs, however, in the following important features which mark it out as a new species: the position and size ratio of the suckers, the presence of an oesophagus, the more or less straight and uniform breadth of the intestinal caeca ending in front of the vitellaria and the subterminal position of the excretory opening.

**Systematic discussion on the genus *Halipegus* Looss 1899 with remarks on the family *Halipegidae* Poche 1925, and the genus *Vitellotrema* Guberlet 1928.**

The systematic position of the genus *Halipegus* has been much debated upon by various workers. Looss, who created the genus in 1899, assigned it a place near the *Syncoeliinae*. Lube in 1901 included it in the family *Hemiuridae*. Ward and Whipple in 1918 and Nicoll in 1926 placed it in the category of unclassified genera. Dollfus in 1923 and Viana in 1924 assigned it to the *Syncoeliinae*. In 1925 Poche, however, created for it a new family *Halipegidae* which he included in his superfamily *Hemiuroidea*. Guberlet in 1928 and Faust 1930 following Poche have maintained the family *Halipegidae*. Odhner in 1927 created a new subfamily *Derogenetinae* under the family *Hemiuridae*, for the genera *Halipegus*, *Derogenes*, *Gonocerca* and *Lecithophyllum* which he considered to be closely related. Fuhrmann in 1928 follows Odhner in assigning the genus *Halipegus* to the *Derogenetinae*.

The genera *Halipegus* and *Derogenes* are closely related on account of the marked similarity in the general body-form, position and size of suckers, length of the intestinal caeca, topography of the gonads and the vitellaria, position of genital pore, large size of eggs with a polar filament at the posterior end and in the excretory system. The only points of difference between the two genera are in the position and arrangement of uterine coils and the extent of the prostate glands—characters which can at the most be considered of generic importance. I, therefore, drop the family *Halipegidae* Poche 1925 and include the genus *Halipegus* in the *Derogenetinae*.

The genus *Vitellotrema* as included in the family *Halipegidae* by Guberlet differs from the type genus of the family only in the unlobed character of the vitelline glands. There is one species of *Halipegus*, i.e., *H. kessleri* syn. *H. rossicus* which has got unlobed vitelline glands like those of the genus *Vitellotrema*. It seems that Guberlet was not aware of the latter condition, as appears from the list of references given in his paper, otherwise he would not have thought of creating his new genus on the basis of this character.



The lobed or unlobed character of the vitelline glands, as discussed by Looss in 1901 and Manter in 1926, should not be considered to be of generic importance even in cases where the lobes are distinctly separated into closely aggregated follicles. This view is also supported by the condition of the vitelline glands in the new species of *Progonus* Looss and of *Ophiocorchis* n. gen. which are described by me in this paper. These species resemble each other closely in most features except in the lobed or unlobed character of the vitellaria. I, accordingly, drop the genus *Vitellotrema* and refer its type species to the genus *Halipegus*.

The diagnosis of the genus *Halipegus* as now constituted is as follows :--

*Derogenetinae*: with a highly muscular and smooth, usually cylindrical rarely flattened body. The suckers are well developed and muscular; the acetabulum larger than the oral sucker, situated about or in the middle of the body. Muscular pharynx present; oesophagus present or absent, intestinal caeca long extending either up to the extreme hinder end or stopping in front of the vitelline glands. The excretory bladder is Y-shaped with a long median stem and two long cornua which run forwards and unite together in the region of the oral sucker or the pharynx. The genital pore is situated either in the region of the pharynx or distinctly behind the intestinal bifurcation; a small genital atrium is present. A ductus hermaphroditicus may be absent or present. The testes two in number, situated symmetrically or asymmetrically in the first half of the post-acetabular region; a small vesicula seminalis and a slight pars prostatica are present but a cirrus is absent. The rounded ovary is situated near the hinder end of the body in front of the vitellaria. The vitellaria lie in two lobed or unlobed groups placed symmetrically or obliquely behind the ovary at the hinder end of the body. Receptaculum seminis is absent. Laurer's canal is present. The long uterus consists of only ascending part in transverse coils containing a huge number of large-sized eggs bearing a long or short polar filament at their posterior end. Parasitic in the mouth cavity, eustachian tubes, pharynx, stomach and intestine of fishes, frogs and snakes.

#### Key to the species of the genus *Halipegus* Looss.

- |  |   |                       |
|--|---|-----------------------|
| Vitelline glands lobed   | A |                       |
| Vitelline glands unlobed   | B |                       |
| A. Testes situated far behind the acetabulum, close to the ovary       |   | <i>H. ovocaudatum</i> |
| Testes situated close behind the acetabulum, far in front of the ovary |   | 1                     |
| 1. Oesophagus present  | 2 |                       |
| Oesophagus absent  | 3 |                       |

2. Intestinal caeca extend up to the extreme posterior end and the excretory pore terminal . . . *H. occidualis*  
Intestinal caeca end in front of the vitellaria, excretory pore subterminal . . . *H. spindale* n. sp.
3. Genital pore lies in the region of the pharynx, the uterine coils do not overlap the intestinal caeca anteriorly in front of the testes . . . *H. longispina*  
Genital pore situated behind the intestinal bifurcation; uterine convolutions extend to the body-wall both in front and behind the acetabulum . . . 4
4. Size 3.1–5.1 mm; acetabulum situated in the middle of the body 1.8 times the size of the oral sucker . . . *H. mehransis* n. sp.  
Size 1.9–1.9 mm; acetabulum situated between first 1/3 and 1/2 of the body and twice the size of the oral sucker . . . *H. mehransis* var. *minutum* n. var.
- B. Intestinal caeca reach behind the vitellaria up to the extreme hinder end of the body . . . *H. fusipora*.  
Intestinal caeca stop in front of the vitellaria . . . *H. kessleri*.

#### Genus *Progonus* Looss 1899 (= *Genarches*)

The only hitherto known species of this genus was described by Levinsen in 1881, for which Looss in 1899 created the genus *Progonus*, assigning it to the *Syncoeliinae* Looss. Luhe founded the family *Hemiuridae* in 1901 and included in it the genus *Derogenes* along with the genera with tail appendage. Looss in 1907 limited the scope of the family and retained under it only such forms as possess a tail appendage. Odhner in 1911 pointed out that *Derogenes* is so closely related to the other *Hemiuridae* that its separation from the family is impossible and that the genus *Progonus* which is closely related to *Derogenes* should be included in the *Hemiuridae*. Nicoll in 1913 agreed with Odhner in this view reducing the family *Hemiuridae* Looss to the position of a subfamily. Ozaki in 1925 described a new genus *Genarchopsis* a form closely resembling *Genarches mulleri* (Levins) and assigned it to the subfamily *Syncoeliinae*. Odhner in 1927 pointed out that *Progonus* shows a close relationship with *Derogenes* in most of its characters and consequently he included it with *Derogenes* in a new subfamily *Derogenetinae*. The only feature in which *Progonus* differs from *Derogenes* is the presence of a caudal anastomosis of the intestinal caeca near the hinder end of the body, which Odhner considers to be an example of "convergence". In the following year Fuhrmann following Odhner included the genera *Derogenes*, *Genarchopsis*, *Gonocerca*, *Licithophyllum*, *Bunocotyle*, and *Halipegus* in the subfamily *Derogenetinae*.

The systematic discussion at the end of the description of the new species of *Progonus* in this paper will show that the genus *Genarchopsis* Ozaki 1925 is identical with *Progonus* and that *P. ovocaudatum* is an intermediate species between the two synonymous genera.

***Progonus piscicola* n. sp.**

(Fig 4)

Host—*Ophiocephalus punctatus*.

Habitat—Stomach.

Locality—Allahabad.

Three specimens of this trematode were obtained from the stomach of one out of about a dozen fish examined in June 1932. In the living condition the parasites are light brown in colour and show great power of contraction and expansion. The body is muscular and somewhat cylindrical in form with a broadly rounded off anterior and a pointed posterior end. The distomes are of moderate size measuring 3·3–3·4 mm in size and 1·12 in maximum breadth which is attained about the middle of the body. The body in front of the acetabulum is uniformly broad while the post-acetabular portion tapers sharply to the posterior pointed end. The well-developed and muscular suckers have a circular outline. The oral sucker measuring 0·33–0·34 mm in diameter lies subterminally at the anterior end of the body, with its cavity directed towards the ventral surface. The acetabulum of 0·66–0·68 mm diameter is twice as large as the oral sucker, situated in the first half of the post-equatorial region.

The oral sucker opens posteriorly into a spherical thick-walled pharynx of 0·12–0·14 mm diameter. In the absence of an oesophagus the intestinal bifurcation takes place directly behind the pharynx at a distance of 0·48–0·53 mm from the anterior end. The intestinal caeca have a highly crenated outline and run at first transversely and then turning downwards continue in a wavy course up to the hinder end of the body where they are continuous into each other just in front of the vitellaria.

The excretory bladder is Y-shaped consisting of an unbranched median stem which bifurcates just behind the acetabulum into two long cornua extending laterally right up to the level of the pharynx and uniting with each other on the dorsal side of the latter. The excretory bladder opens terminally at the hinder end of the body. The terminal part of the bladder is surrounded by a sphincter formed by a group of deeply staining parenchymatous cells with prominent nuclei.

The semilunar slit-like genital pore is sinistral or median, situated ventrally in level with the pharynx. It leads into a roomy genital atrium of

left vitelline gland measures  $0.13-0.19 \times 0.05-0.08$  mm in size while the right one is of  $0.08-0.17 \times 0.09-0.1$  size.

This interesting species resembles *P. piscicola* n. sp. in the general body-form and size ratio of the suckers, absence of prepharynx and oesophagus, the end apparatus of the reproductive system and the lateral extent of the uterine coils. It differs, however, from the above species in the smaller size of body, position of the acetabulum, the course of the intestinal caeca, more caudal position of the testes, smaller size and position of the shell gland mass, the symmetrical position of the vitellaria and in the important fact that the uterine convolutions extend posteriorly beyond the shell gland mass and lie in the space between the two compact vitelline glands at the extreme hinder end of the body. In this last character this species resembles *P. mulleri*. (Levins.)

#### **Systematic discussion on the genus *Progonus* Looss (= *Genarches* Lss.) and on the genus *Genarchopsis* Ozaki 1925**

In discussing the diagnostic features of his new genus "*Genarchopsis*" Ozaki points out that his species bears a very close resemblance to *Genarches mulleri* (Levins) in the general shape of the body, the structure of the end apparatus of the reproductive organs and the excretory system but it differs in the convolutions of the uterus which do not stretch back further than the vitellaria. We find that *P. ovocaudatum* n. sp. resembles *Genarchopsis goppo* in nearly all features except in the extent of the uterus which extends further backwards than in the latter species, reaching up to the hinder end of the vitellaria or body. As the only distinction between the genera *Genarchopsis* and *Progonus*, i.e., in the extent of the uterus ceases to exist in my species *P. ovocaudatum*, the identity of *Genarchopsis* with *Progonus* becomes quite clear. I maintain that these two genera are identical and synonymous. This view is also supported by the condition of the uterine coils in the two species of *Ophiocorchis* n. gen. described in subsequent pages.

I assign the genus *Progonus* to the subfamily *Derogenetinae* Odhner 1927. I agree with Odhner that the posterior anastomosis of the intestinal caeca in *Progonus* must not be given undue systematic importance. The posterior intestinal anastomosis is present in many distantly related forms such as *Cyclocalium*, *Progonus*, *Opicoelous* and *Coitcoecum*, etc., and therefore must be considered as an example of convergence.

In view of what has been said above the diagnosis of the genus *Progonus* as given by Looss in 1899 needs a certain amount of modification. The amended diagnosis is as follows:—

Small distomes with elongated, flattened or cylindrical body tapering at both ends; suckers strongly developed; skin entire. Prepharynx present, oesophagus absent, intestinal caeca continuous into each other at the hinder

end of the body. Genital pore situated near the hinder end of the pharynx or behind the intestinal bifurcation; genital sinus is formed by the union of the male and female ducts and opens on a genital papilla; pars prostatica and the vesicula seminalis, however, not strikingly elongated and the latter does not reach the acetabulum towards its hinder end. Testes and ovary simple, rounded or oval. The vitellaria consist of two glands which may be lobed or compact, lying one on either side close behind the ovary. The uterine convolutions may or may not stretch back up to the hinder end of vitellaria. Excretory system as typical of the subfamily, *i.e.*, Y-shaped with the cornua uniting dorsal to the oral sucker or pharynx. Parasitic in the intestine and stomach of fresh-water and marine fishes.

### Key to the species of the Genus *Progonus* Looss 1899

- Uterus extends behind the shell gland mass reaching up to the posterior part of vitellaria ... 1
- Uterus does not extend posteriorly up to the vitellaria ... 2
- 1. Genital pore situated at the hinder end of pharynx ... *P. mulleri*.
- Genital pore situated a little behind the intestinal bifurcation ... *P. ovocaudatum*.
- 2. Uterine coils confined to intercaecal space ... *P. goppo*.
- Uterine coils not confined to the intercaecal space but extending to the bodywall on either side ... *P. piscicola*.

### *Ophiocorchis lobatum*. n. gen.; n. sp.

(Fig. 6)

Host—*Ophiocephalus striatus*.

Habitat—Stomach.

Locality—Lucknow.

In September 1932 I examined about thirty living specimens of *O. striatus* received from Lucknow, only two of which were found infected with two specimens each of this parasite. The distomes have a highly muscular and cylindrical body with smooth cuticle. In permanent mounts they measure 2·8–3·2 mm in length and 1·1–1·2 mm in breadth across the acetabular region. The well-developed suckers have a spherical outline. The oral sucker, 0·3 mm in diameter, occupies a subterminal position on the ventral surface. The acetabulum, measuring 0·76 mm in diameter, is 2·5 times the size of the oral sucker and is situated about the middle of the body.

Terminally the uterus passes into a well-developed muscular metraterm of  $0.15 \times 0.05$  mm size which after receiving the pars prostatica is continued as the ductus hermaphroditicus of 0.12 mm length. The ductus hermaphroditicus is capable of being protruded out of the 0.075 mm deep genital atrium and functions as a copulatory organ. The eggs measure  $0.035 \times 0.017$  mm in size and bear a polar filament of 0.012 mm length at one end.

*O. singularis* n. sp. bears a very close resemblance to the type species *O. lobatum* n. sp. The points of similarity are the presence of an oesophageal pouch, a metraterm, a pars prostatica and the topography of the gonads. It differs, however, from the latter species in the smaller size of the body, the extent of its uterine coils, position and size of the acetabulum and the compact nature of the vitelline glands.

I assign this new genus *Ophiocorchis* to the subfamily *Derogenetinae* of the family *Hemiuridae*. Of all the genera of the subfamily this interesting parasite in its affinities comes nearest to the genus *Progonus* Looss 1899 (as already amended by me in this paper). It shows some resemblance with the latter genus in the general body-form, position of the genitalia and the arrangement of the uterine coils. The remarkable points of difference which warrant the creation of a new genus for its reception are the presence of a well-developed globular pars prostatica, a large and highly muscular metraterm, a protrusible ductus hermaphroditicus capable of functioning as the copulatory organ and the presence of a peculiar structure which I have termed as the oesophageal pouch.

#### Key to the species of the genus *Ophiocorchis*

1. Vitelline glands lobed and the uterine coils do not extend in the region of the vitellaria ... *O. lobatum*.
2. Vitelline glands compact with the uterine coils extending between the two compact vitelline glands ... .. *O. singularis*.

#### EXPLANATION OF THE PLATES

- Fig. 1. Ventral view of *Halipegus mehransis*.  
 Fig. 2. Ventral view of *H. mehransis* var *minutum*.  
 Fig. 3. Ventral view of *H. spindale*.  
 Fig. 4. Ventral view of *Progonus piscicola*.  
 Fig. 5. Ventral view of *P. ovocaudatum*.  
 Fig. 6. Ventral view of *Ophiocorchis lobatum*.  
 Fig. 7. Ventral view of *O. singularis*.  
 Fig. 8. Diagrammatic view of female sexual organs of—*H. mehransis*.  
 Fig. 9. Diagrammatic view of female sexual organs of—*P. piscicola*.

## LETTERING

Act.	...	Acetabulum	Oot.	...	Ootype
C	...	Cirrus	Ph.	...	Pharynx
C. s.	...	Cirrus sac	P. ph	...	Prepharynx
Eg.	...	Egg	Pr. g.	...	Prostate glands
D. ej	...	Ductus ejaculatorius	P. p.	...	Pars prostatica
D. h.	..	Ductus hermaphroditicus	R. s.	...	Receptaculum seminis
E. b.	...	Excretory bladder	R. s u.	...	Receptaculum seminis uterinum
E. p.	...	Excretory pore	S. gl.	...	Shell gland
G. a.	...	Genital atrium	S. gl. c.	..	Shell gland complex
G. p.	...	Genital pore	T.	...	Testis
Int. c.	...	Intestinal caecum	Ut.	...	Uterus
L. c.	...	Laurer's canal	V. d.	...	Vas deferens
Mtm.	...	Metratrem	Vit.	...	Vitellaria
Oes.	..	Oesophagus	V. sm.	...	Vesicula seminalis
Oes. p.	...	Oesophageal pouch	V. s.	...	Ventral sucker
O. s	...	Oral sucker	Y. d.	...	Yolk duct
Ov.	...	Ovary	Y. r.	...	Yolk reservoir
O. d.	...	Oviduct			

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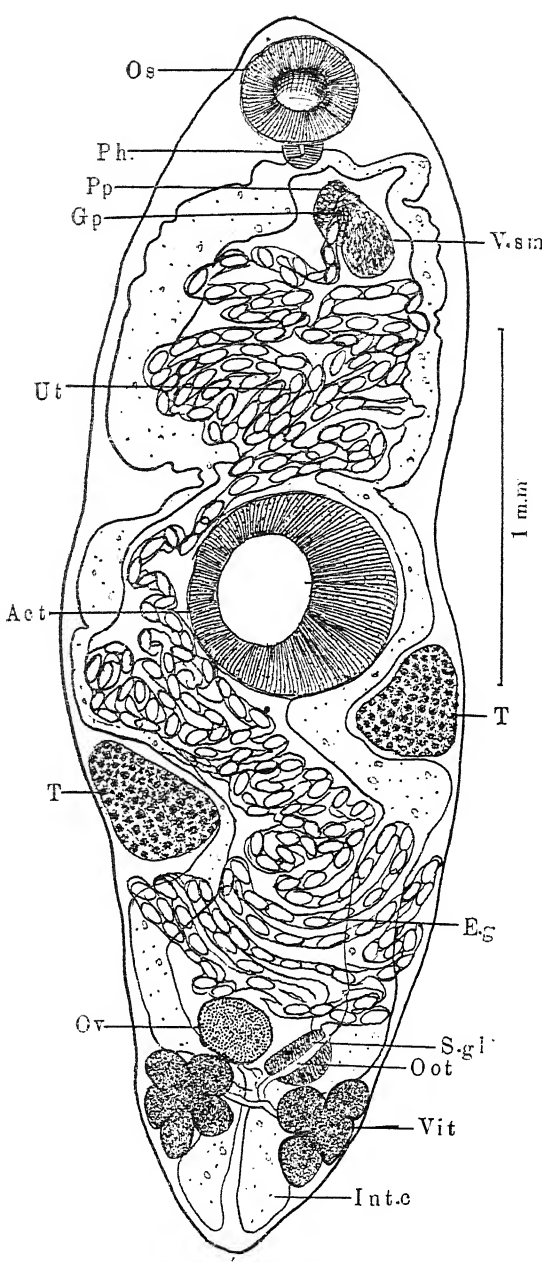


FIG. 1.—*Halipegus mehransis*, n. sp.

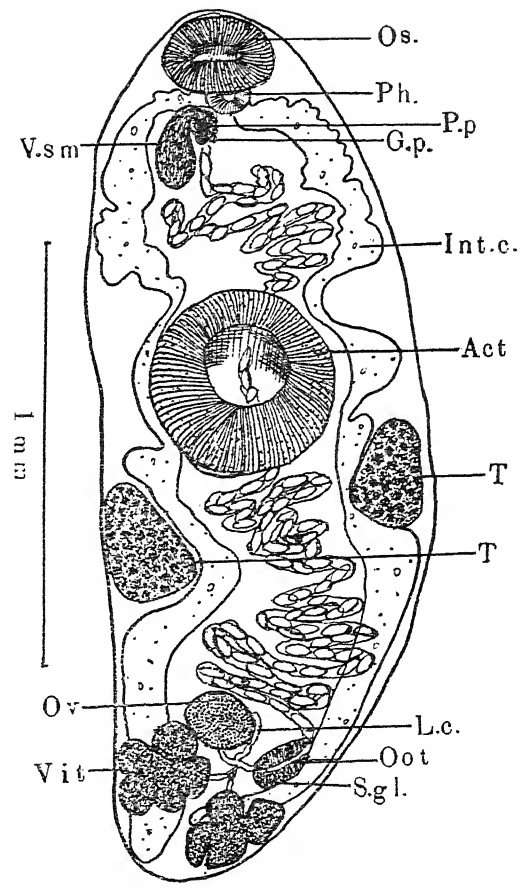


FIG. 2.—*Halipegus mehransis*, var *minutum*.

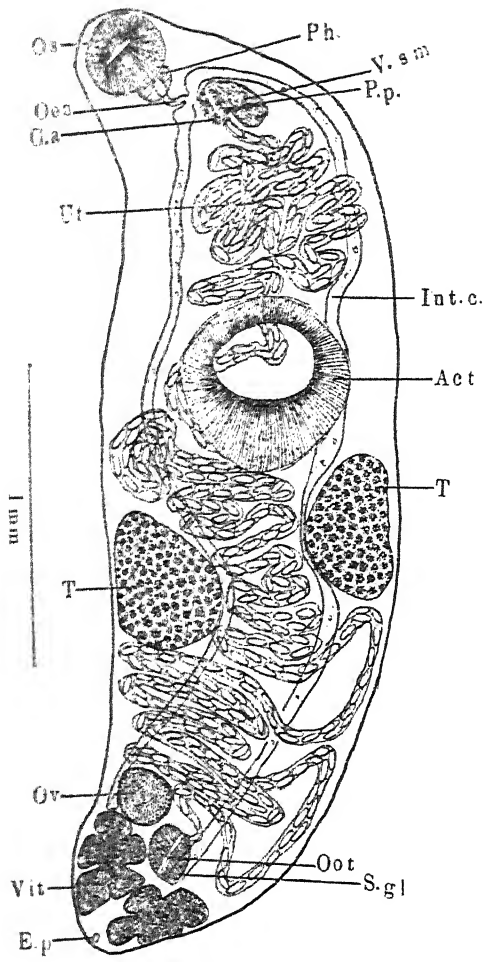


FIG. 3.—*Halipegus spindale*, n. sp.

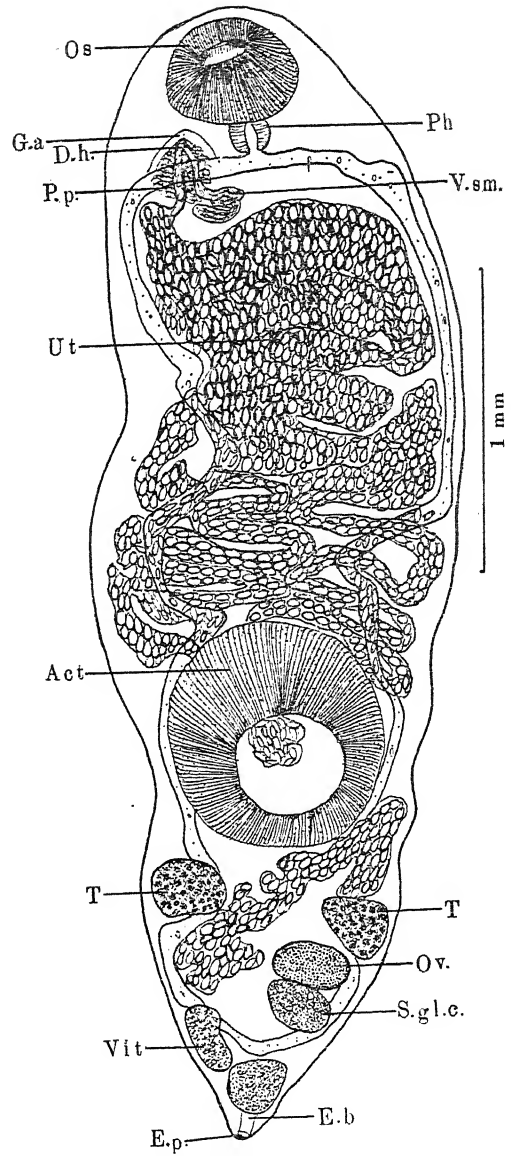


FIG. 4.—*Progonus piscicola*, n. sp.

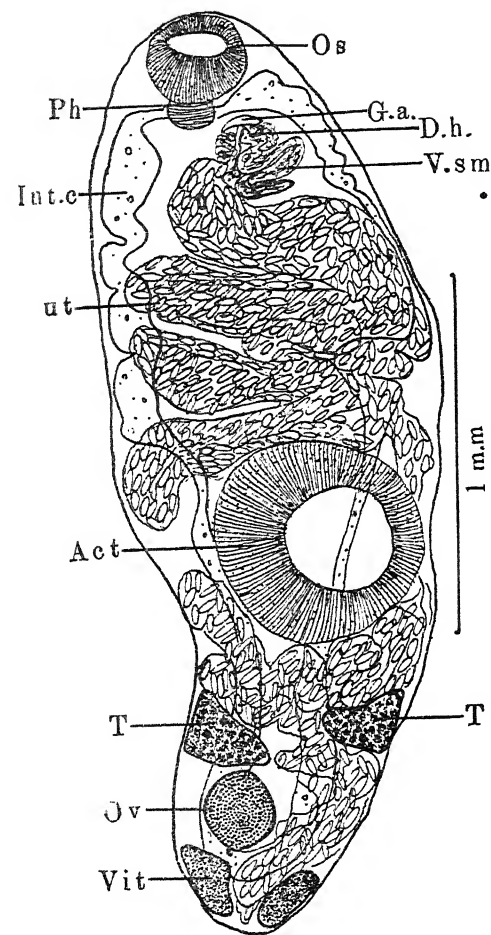


FIG. 5.—*Progonus ovocaudatum*

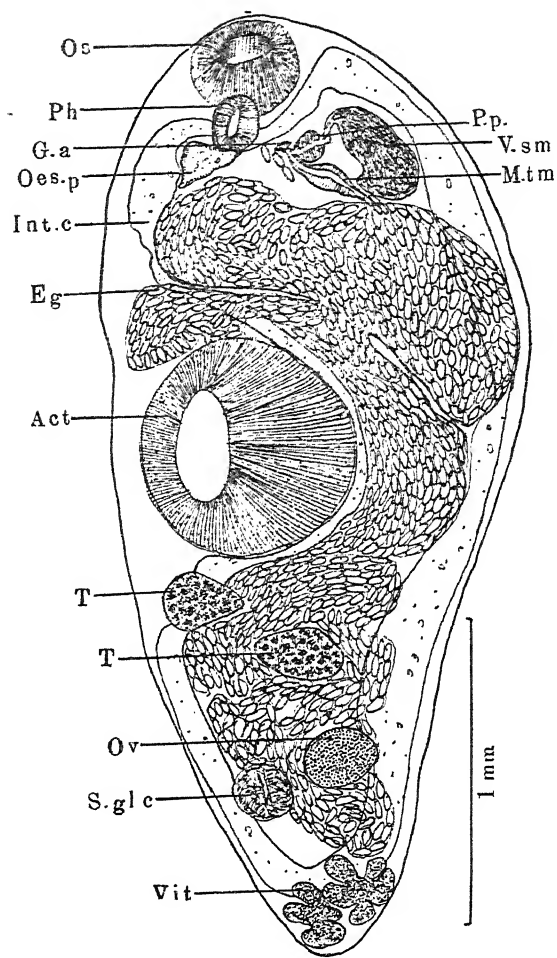


FIG. 6.—*Ophiocorchis lobatum*, n. gen., n. sp.



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